Search and switching costs

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Publisher: University of Groningen, Groningen, The Netherlands

Printer: Ipskamp Printing B.V.

ISBN: 978-90-367-8467-2 / 978-90-367-8466-5 (eBook)

This document was prepared using LaTeXusing Peter Dijkstra's much appreciated style file.

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Search and Switching Costs

Proefschrift

ter verkrijging van de graad van doctor aan de
Rijksuniversiteit Groningen
op gezag van de
rector magnificus prof. dr. E. Sterken
en volgens besluit van het College voor Promoties.

De openbare verdediging zal plaatsvinden op

donderdag 14 april 2014 om 16.15 uur

door

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geboren op 25 februari 1987
te Emmen

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Acknowledgements

Wuthering Heights, War and Peace, Les Miserables, 1984, Hamlet, Great Expectations; these are among the greatest works produced by humanity over time. This book does not fit in this list, as it is scientific (amongst many other reasons). I would like to claim that, given my humble capabilities, the achievement of completing this thesis is bigger than writing the books mentioned, but I gratefully admit I had immense support of many colleagues, friends and family during the process.

First I would like to thank the dream-team of supervisors which supported me during the last couple of years, consisting of Marco Haan and Rob Alessie. Although their research capabilities are of extraordinary level beyond any doubt, Rob and Marco also proved to be nice and good persons outside the work environment. Both of them gave me a lot of freedom and allowed me to pursue my own research interests but also provided guidance when needed.

I met Marco during the master-stage of my studies and he has taught me many things about economic theory ever since. We had many vivid discussions which always provided renewed inspiration for research. Moreover, Marco showed me how to write and tell stories instead of presenting a bunch of equations. He ensured I kept my eyes on the bigger picture and that I would not lose myself too much in a set of mathematical puzzles which were, although interesting, less relevant. At about the same time I met Rob for the first time. Rob introduced me in an understandable way to a wide set of complicated econometric models. I admire how friendly and helpful Rob always is and how in the meantime he has been able to become a researcher with an incredible track-record. Although some topics in this thesis are not directly related to his field of expertise, his office door has always been open and he has given me invaluable advise on some important issues.

The reading committee, consisting of Yongmin Chen, José Luis Moraga-González and Adriaan Soetevent, also deserves gratitude for carefully reading the thesis and providing their helpful comments. I also like to thank the people of the SOM office and the secretaries, especially Ellen and Martine, for taking care of numerous organizational issues during my PhD.

Besides those mentioned above I would like to thank all the people from the 7th floor of the WSN building for providing a great working environment. Some of them even helped to provide the basis for this thesis by lecturing me during my bachelor-studies, for which I am grateful. In that period I spent numerous hours in classrooms and the company of Richard, Niels, Gianna, Anke and Bram made it a wonderful experience.

I also would like to thank all my former office-mates: Remco, Umar, Dominic and Pieter for interesting interactions, challenging riddles, a perfect working environment, and not bringing *rijstwafels* to the office. All of them stayed for about a year and then graduated. This leaves me wondering whether my presence encouraged them to leave quickly or whether I possess qualities similar to Richard Neville, the King-maker.

A special thanks goes to Dominic, who had enough confidence in me to entrust me with the office plant and with whom I spent numerous hours in the ACLO, on the football pitch and in pizza-joints. Later on Sebastian joined me on these kind of trips on which he taught me some important Polish words.

I also would like to thank my friends Nikita, Rick, Jacob, Shu, Yan and (again) Dominic with whom I enjoyed the tea-breaks. We had many lively discussions about politics, research, football, life, philosophy and nonsense, topics which are not necessarily mutually exclusive.

Although I know a PhD-student is supposed to have no social life outside academia, I still would like to show my gratitude to the many people out there who contributed in various ways to this thesis.

First, I like to thank my handball-friends. My time at V&S was brilliant, I played for various teams but not one can match this weird cross-section of society which practiced with Menhirs, matts and hairbands in order to do some activity involving a bear in the weekend.

Second, thanks goes to my current colleagues at Menzis where I felt instantly welcome, which enabled me to write the last part of this thesis. Especially my admittance to the bowling-team and the CVHJ-subleague are highly appreciated.

Third, a great thanks goes to Maarten, Michiel, Guus, Geert and Paul with whom I made some legendary trips through the years and reminded me that there is still consumer behaviour that cannot be explained by the existing economic theories.

Finally, my family deserves a special mention in these acknowledgements, they have always provided an environment which allowed me to explore, develop and grow. Between me and my brothers there exists a healthy rivalry to outperform each other. Rick, Niek and Annie, and Tim have always supported and helped me in all kinds of ways despite this: thank you for that. I am most grateful for my parents, Theo and Catrien. Thank you for your unconditional support and love. I am proud to be a member of this family.

Wilhelm Henricus (Wim) Siekman

February, 2016

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Part II: Switching

Chapter 1

Introduction

Background

Economics is the science that studies the allocation of scarce resources. Many economic models have been developed to investigate how to distribute resources more efficiently. However, only since the middle of the second half of the last century (e.g. Diamond 1970) it has been realized that there are important frictions on the market that may lead to inefficiencies, failure, or a significant redistribution of welfare. This thesis focuses on (two of) these frictions, search and switching costs, and their interactions with other economic phenomena such as behavioral biases and retention offers.

Search costs are the costs consumers have to incur in order to obtain information about a product and its price. These exist in many forms, such as fuel costs that have to be incurred to visit a shop, time spent reading a product description, or energy and internet costs paid to browse websites. Switching costs, on the other hand, are costs to switch from one supplier to another once the product and its price are fully known. These might exist in the form of a fine for terminating the current contract prematurely or as the administrative cost and effort to inform other parties about your new phone or bank account number. Another interesting example of switching costs exists in the marriage market, as switching from partner may lead to loss of contact with children or income.

The aim of this thesis is to provide insights in the mechanisms that are active in markets with search and/or switching costs. This thesis consists of, besides this introduction, four chapters. Each of these is self-contained and therefore there is no need to elaborately discuss them here. Instead I will give a short overview of their aims, results, and contributions to the literature.

Chapter 2. Search when Consumers Are Loss Averse

Chapter 2 studies the interaction between search costs and loss aversion. Loss aversion is a behavioral bias that was first identified in the seminal paper of Kahneman and Tversky (1979). The concept of the bias is that if outcomes differ from a reference point (often based on the consumer's expectations) then disappointments yield additional disutility, while pleasant surprises provide additional positive utility – but at a lower rate.

A small part of the behavioral economics literature, which studies to what extent competitive firms can take advantage of behavioral biases and how market performance is affected as a result, already focuses on loss aversion. Prominent examples are Heidhues and Köszegi (2008) and Karle and Peitz (2014). In these papers, consumers are disappointed if they end up with a product that is further from their most preferred product than what they *ex ante* expected. Also, they experience a disutility if they observe a price that is higher than the tentative equilibrium price. In turn, this affects that equilibrium price.

In this body of work it is assumed that all products are readily observable. Consumers thus have to simultaneously evaluate all the products on offer, and hence they also have to evaluate feelings of gain or loss regarding those offerings simultaneously. A more natural environment would allow consumers to evaluate offers from firms sequentially. A consumer may then be disappointed if the product that she is currently considering or its price is worse than expected, and may decide whether to evaluate the next product on the basis of that.

Chapter 2 exactly models such an environment. In a framework of costly consumer search with differentiated products, it introduces consumer loss aversion. The chapter builds on the seminal papers of Wolinsky (1986) and Anderson and Renault (1999) in which consumers sequentially search firms to find one that offers a product that is sufficiently to their liking. In such setting consumers are allowed to feel an additional loss whenever they encounter a product that is more expensive or that has a lower match value than what they expected to find. The chapter also allows for the possibility that a consumer experiences additional gains if she finds a product with a better match value or price than expected.

Loss aversion is found to lead to a range of possible Nash equilibria, as is also the case in e.g. Heidhues and Kőszegi (2008). As in their work, the upper bound on equilibrium prices increases in the extent of loss aversion, but different from their results the lower bound might decrease. Hence, loss aversion may lead to lower prices.

In the model, loss aversion may affect consumers in three dimensions. First, loss aversion affects consumers in the price dimension: consumers are disappointed when they encounter a price that is higher than expected. When viewed in isolation, this channel leads to weakly lower prices, as it makes firms more reluctant to increase prices. Second, loss aversion affects consumers in the search dimension: they may be disappointed when they have to search more often than expected. When viewed in isolation, this leads to unambiguously higher prices since it effectively increases search costs. Third, loss aversion affects consumers in the match value (or product preference) dimension. In isolation, this leads to unambiguously higher prices as well, as the range of possible match values is now higher, leading effectively to more product differentiation and hence higher prices. In addition loss aversion is found to yield more search when search costs are relatively low, but less search when they are relatively high.

Taken together, the total effect of loss aversion on prices is ambiguous. Therefore the chapter introduces an equilibrium refinement which essentially allows consumers to make infinitesimally small mistakes with respect to the Nash equilibrium price on which market participants coordinate. The unique price that survives this refinement is decreasing in the extent of loss aversion for intermediate search costs and relatively high loss aversion, and increasing otherwise.

Chapter 3. Directed Consumer Search

On many markets there are multiple suppliers who offer products which differ in several dimensions. To find the best offer on these market consumers will have to search. They can search in a random way, but they can also rely on intermediating parties such as search engines and online marketplaces to obtain information on which seller offers a product and deal to their liking. For example, if a consumer wants to buy a new computergame, shopping websites suggest titles based on her order history. Besides entering a product name into a search engine or comparison website people can also specify some features they like. It is clear that internet allows the search order to depend upon one's preferences, however, even offline this possibility exists. A person looking for some classical literature will not be directed to the comics section in a bookstore, and a person looking to buy a car with high MPG will not start searching at a car dealer who primarily advertizes SUV's.

New insights are offered in chapter 3 by allowing consumer preferences to affect the listing order on intermediating platforms, and therefore the search process of consumers. In the model that is presented in that chapter a product consists of so-called communicable and incommunicable horizontal characteristics. For instance, consider a consumer wanting to rent a house. She visits a platform in order to find a property to her liking. On the platform she might, for example, indicate that she prefers a house in close proximity to her new workplace, this is a communicable characteristics. The platform suggests, based on this information, the real-estate agent at which the consumer is most likely to find the most suitable house. Hence, the consumer is *directed* in her search for the best house. In turn the consumer visits the website of suggested real-estate agent in order to find out the price and inspect the product's incommunicable characteristics, such as whether the house's layout is convenient. She can then rent the property or decide to continue her search until she does find an offer to her liking.

Prices and profits are found to be higher when consumers are directed in their search. The reason is that sellers exploit the fact that consumers are less likely to visit a competitor when they first visit the seller with the best communicable attributes. In effect, when product characteristics are communicated products are ex-ante less homogeneous for consumers. This result on prices is opposite to that of models in which one seller is made prominent and all consumers sample this seller first. There the prominent seller sets a lower price as compared to the case of random search as she faces less elastic demand. The other sellers, on the other hand, face more elastic demand and therefore charge higher prices.

Chapter 3 considers the welfare effects of search as well. On average consumers obtain a better match when product information is used to determine the search order, since they first visit the seller where they are most likely to find the product that matches their preferences the best. In addition, consumers are more likely to cease their search in each stage of the process. The reason is that consumers realize that the first seller they visit offers a product which is, for certain characteristics, the best on the market. Therefore consumers spend less on search costs. Because of these two effects the total welfare is higher under the alternative search strategy compared to the case of random search. Under the alternative search strategy, however, consumers are worse off as these two effects are outweighed by the higher prices they have to pay.

Again it is interesting to compare this result with models in which one seller is made prominent. There consumers also search less, although for a different reason. In those models the first seller sets a lower price than the competition and consumers are therefore more likely to buy there. This means that some consumers do not search beyond the first seller although it would be socially optimal. Hence, in those models consumer obtaion on average a lower match value than in case of random search. This implies that in those models non-random search leads to lower total welfare while in my model it has the opposite effect.

The result that welfare is raised by directing consumers in their search has one important implication. Consider a profit maximizing platform that provides a link between consumers and sellers. If, by charging both consumers and sellers, the platform is able to extract total welfare it will have an incentive to suggest different sellers for different groups of consumers depending on consumer's preferences.

Chapter 3 extends the seminal work of Anderson and Renault (1999) in which consumers visit sellers in a random order. By introducing communicable characteristics I allow the search order to be influenced by consumer preferences. The model of Anderson and Renault (1999) can be obtained from the one presented in Chapter 3 by allowing a product only to consist of incommunicable characteristics. However, in the other extreme, when all product characteristics are available to consumers prior to search, the paradox of Diamond (1971) results. My work inherits the result from Anderson and Renault (1999) that prices are non-decreasing in search costs.

Chapter 3 contributes to two strands of literature. First, it extends to the literature on consumer search with differentiated products, in which the classic papers by Wolinsky (1986) and Anderson and Renault (1999) play a central role. These and most subsequent works assume that consumers search at random. An exception close to my work is Armstrong et al. (2009). In this paper one seller in the market is allowed to be prominent, and this seller is always sampled first. This a-symmetry is not imposed in the model of chapter 3 and, a priori, every seller is equally likely to be visited first by the consumer. Moreover, the consumer's preferences are allowed to influence the search order. These subtle differences in modeling leads to some interesting opposite conclusions.

Second, chapter 3 also connects to the literature on position auctions. Works in this field typically consider sellers who differ in the likelihood that they can meet the 'need' that a consumer has. The focus of this literature is on how the platform can maximize profits by auctioning prominent positions to sellers but it often abstracts away from the relationship between sellers and consumers.

Chapter 4. Winning back the unfaithful while exploiting the loyal: Retention offers and heterogeneous switching costs

In subscription-type markets, e.g. those for credit cards, telecom, and insurance, firms are often willing to make so-called retention offers: a better deal to consumers who indicate that they want to cancel their subscription. Consumers' reaction to these practices differ. Some of them seem largely unaware of it, or at least unwilling to exploit such offers, while others actively chase them.

In Chapter 4 retention offers are analysed in a setting with two types of consumers; those with relatively low, and those with relatively high switching costs. Retention offers can be used by firms to screen consumers with low switching costs. Consumers that have already gone through the trouble of obtaining an offer from a competing firm, signal that they have low switching costs and therefore are likely to switch. Retention offers then effectively provide a mechanism to price discriminate against consumers with high switching costs.

The chapter thus focuses on cases where consumers cancel their current subscription in favor of a competitor. For example, in mobile telephony in the UK, consumers that want to switch have to contact their current provider and request a code which must be communicated to their new provider to complete the switch, see Ofcom (2010). However, when applying for such a code, the current provider often makes a retention offer.

A two-period model is studied with two firms located at the endpoints of a Hotelling line. In the second period, firms set prices based on consumer behavior in period 1. In particular, firm B can charge consumers from firm A a lower price in order to try to poach them. Once a consumer indicates that she intends to switch from A to B to take advantage of that poaching price, however, a retention offer can be made by firm A. A profitable strategy for low-switching-cost consumers, in the equilibrium of the model, is to solicit offers from the competing firm to obtain a retention offer from their current provider – even if they do not intent to switch. Securing retention offers requires costly effort, and high-switching-cost consumers do not find making that effort worthwhile. Hence, firms can use retention offers to price discriminate between the two types of consumers.

Chapter 4 fits in the literature on behavior-based price discrimination and the literature on switching costs. However, studies on retention offers are scarce. Papers that do study this subject either consider, different from this chapter, a market of homogeneous goods, or do not allow consumers to strategically solicit an offer from a competing firm, in an attempt to obtain a better deal from their current supplier.

The analysis in Chapter 4 suggest that the possibility of retention offers increases prices. Prices for loyal consumers increase, as these consumers are less likely to switch on average. But poaching prices increase as well; low-cost consumers become easier to poach as they already have incurred part of their switching costs. Prices in the first period increase as well. As competition for consumers with low switching costs is fiercer in the second period, firms are less eager to attract these consumers in period 1. The welfare effects of the possibility retention offers are ambiguous. Consumers are worse off, while firms obtain higher profits. The former applies to all individual high-switching-cost consumers, and to consumers as a whole. The effect on the welfare of individual low-cost consumers is ambiguous.

Chapter 5. Marital infidelity and impatience: Infidelity and smoking habits

Finally, chapter 5 studies empirically a market where switching costs are substantial: the marriage market. More specifically, I empirically investigate there the relationship between impatience and the first step in the switch of partner: marital infidelity. Adultery is a subject of substantial importance as it is the cause of many divorces. A divorce can have severe economic consequences for a household such as the loss of income, economics of scale and future pension entitlements. Moreover, a divorce might also affect household members in non-pecuniary ways. It might, for instance, have a negative impact on children's welfare. Even in the case that a relationship survives when an affair is disclosed, the utility derived from it might be lower as the disclosure signals marital dissatisfaction. Additionally, the marital terms might be re-negotiated. Moreover, it is interesting to study adultery as intercourse plays an important role in society along with its associated phenomena like contraception, sexual diseases, abortion and shot-gun marriages.

There exists a small economic literature on the subject of adultery. This literature considers a large set of possible determinants of infidelity, but not many papers take into account that the consequences of an affair should be discounted as they lay in the future. One exception is Smith (2012), who bases his approach upon the literature of economics of crime (e.g. Becker (1968)). His model takes in account that cheating individuals realize that their spouse might punish them once an affair is discovered. This is the rationale for individuals to try to conceal their affairs. Smith (2012) allows an individual's discount factor to depend upon the level of education. However, education is also permitted to affect the infidelity decision in other ways.

In Chapter 5 I extend the theoretical model of Smith (2012) by letting an individual's discount factor to be affected by his level of impatience, and I estimate it. In the empirical analysis smoking behavior is used to measure impatience. Empirically it has been firmly established that smokers are more impatient than non-smokers.

The estimated coefficients on the variables related to smoking are found to be significant. In the literature it is found that persons who (used to) smoke are more impatient. This suggest that the level of a person's impatience affects the likelihood of adultery.

Hence, the discounted expected costs should be accounted for when modeling infidelity.

Some new light is also shed on the relationship between infidelity and education by the analysis. Fair (1978), a seminal work in this branch of literature, found that education is negatively associated with infidelity and occupation positively. As both education and occupation are proxies for wages this is somewhat surprising, and the time allocation framework that Fair (1978) employs cannot explain this. My model and that of Smith (2012) do offer an explanation for the opposing signs of education and occupation through the following mechanisms. Occupation measures a person's quality as a sexual partner and the related benefits from an extramarital affair. It does not capture a person's sexual desirability fully, but still it is clearly positively correlated to his socio-economic status. On the other hand, the likelihood of adultery is affected by education in three ways. Firstly, schooling affects an individual's quality as more educated individuals are able to find better jobs and collect higher wages. This is the argument that Fair (1978) used to justify schooling as a wage proxy. Secondly, education raises a person's social skills. Through these two mechanisms schooling adds to the benefits of an affair and raises a person's desirability. Thirdly, higher educated persons have a lower discount factor for the expected costs of an affair. This provides the reason why infidelity might be negatively correlated with education. Smith (2012) finds, as Fair (1978), the relation between schooling and adultery to be negative. Chapter 5, however, shows that the discounting-effect of education is (partly) filtered out once one controls for the impatience level of an individual. As a result the quality (signaling) effect of education becomes more dominant, and the association between schooling and infidelity becomes more positive. I find that the relationship between schooling and extramarital sex even becomes positive for some of the estimations, which suggests that it is important to control for impatience when modeling infidelity.

Concluding thoughts

To conclude, this thesis provides interesting insights in the effects of search and switching costs, possibly in combination with other economic phenomena such as loss aversion and retention offers. An important finding is that firm activities which might seem to benefit consumers, such as retention offers or communicating product information, actually decrease consumer surplus and increase profits. Moreover, heterogenous switching costs are shown to have a relevant impact on market mechanisms. Chapter 5 shows the empirical effects of the components of these costs, while chapter 4 proves that such heterogeneity offers strategic possibilities. The role of behavioral biases on markets should also not be underestimated, which is apparent from chapter 2 where the amount of loss aversion determines the price level. Economic policy (for markets with search and switching costs), and the models on which this is build, should therefore take these phenomena into account before drawing conclusions.

The remainder of this thesis consist of two parts. The first part focuses on the subject of search costs and presents the above discussed chapters 2 and 3. The other chapters that are described above, chapters 4 and 5, make up the second part of the thesis, which revolves around switching costs. In the appendix a (Dutch) summary of this thesis can be found.

Part I

Search

Chapter **2**

Search when Consumers Are Loss Averse*

2.1 Introduction

Over the last years, a literature has emerged that studies competition in a world where consumers are less than fully rational (see e.g. Spiegler, 2011). Inspired by systematic biases unearthed by the experimental and behavioral economics literature, this literature studies to what extent competitive firms can take advantage of such behavioral biases, and how market performance is affected as a result.

One behavioral bias that has received particular attention is that of loss aversion. If outcomes differ from a reference point (often based on the consumer's expectations) then disappointments yield additional disutility, while present surprises provide additional positive utility – but at a lower rate. This bias was first identified in the seminal paper of Kahneman and Tversky (1979). To formally study the effect of such gain-loss utility in a decision-making framework, Köszegi and Rabin (2006) introduce the concept of personal equilibrium, which requires that an agent's behavior optimally and rationally takes into account the possibility of future gain-loss utility in an intertemporal context.

A small literature has originated that studies the effect of gain-loss utility on the functioning of markets. Prominent examples include Heidhues and Köszegi (2008) and Karle and Peitz (2014). In these papers, consumers are disappointed if they end up with a product that is further from their most preferred product than what they expected *ex-ante*. Also, they experience a disutility if they observe a price that is higher than

*This chapter is based on Haan and Siekman (2015a).

the tentative equilibrium price. In turn, this affects that equilibrium price.

Yet, in this literature it is assumed that all products are readily observable. Consumers thus have to simultaneously evaluate all the products on offer, and hence they also have to evaluate feelings of gain or loss regarding those offerings simultaneously. A more natural environment would allow consumers to evaluate firm offerings sequentially. A consumer may then be disappointed if the product that she is currently considering or its price is worse than what she expected, and may decide whether to evaluate the next product on the basis of that.

In this chapter, we model exactly that. In a framework of costly consumer search with differentiated products, we introduce consumer loss aversion. We build on the seminal papers of Wolinsky (1986) and Anderson and Renault (1999) in which consumers sequentially search firms to find one that offers a product that is sufficiently to their liking. In such a model we allow consumers to feel an additional loss whenever they encounter a product that is more expensive or that has a lower match value than what they expected to find. Moreover, we propose an equilibrium refinement to deal with the multiplicity of equilibria that are routinely found in competition models with loss aversion. That refinement yields a unique equilibrium and is also readily applicable to other competition models with loss averse consumers.

We find that loss aversion leads to a range of possible Nash equilibria, as is also the case in e.g. Heidhues and Köszegi (2008). Yet, different from their work, we find that although the upper bound on equilibrium prices increases in the extent of loss aversion, the lower bound might decrease. Hence, loss aversion may lead to lower prices. We also propose an equilibrium refinement that yields a unique equilibrium. This refinement essentially allows consumers to make infinitesimally small mistakes with respect to the Nash equilibrium on which market participants coordinate.

In our model, loss aversion may affect consumers in three dimensions. First, loss aversion affects consumers in the price dimension: consumers are disappointed when they encounter a price that is higher than expected. When viewed in isolation, this channel leads to weakly lower prices, as it makes firms more reluctant to increase prices. Second, loss aversion affects consumers in the search dimension: they may be disappointed when they have to search more often than expected. When viewed in isolation, this leads to unambiguously higher prices since it effectively increases search costs. Third, loss aversion affects consumers in the match value (or product preference) dimension. In isolation, this leads to unambiguously higher prices as well, as the range of possible match values is now higher, leading effectively to more price discrimination and hence higher prices. The total effect is ambiguous. Loss aversion yields more search when search costs are relatively low, but less search when they are relatively high. The unique price that survives our equilibrium refinement is decreasing in the extent of loss aversion for intermediate search costs and relatively high loss aversion, and increasing otherwise.

In sequential search models like the one we are considering in this chapter, consumers search until they find a match value that is sufficiently high such that continuing search is not worth their while. In our base analysis we assume that consumers take into account that in equilibrium they will end up with a match value that is at least this high. Any lower match value will then be a disappointment. In an extension we study the case in which the entire distribution of match values acts as a reference. In that case, we find the same qualitative results. The unique price that survives our equilibrium refinement is decreasing in loss aversion for small search costs and large loss aversion, and increasing otherwise.

Related literature includes the following. Heidhues and Köszegi (2008) use the approach of Köszegi and Rabin (2006) to study the effect of loss averse consumers on a Salop circle and find that such loss aversion weakly increases equilibrium prices. In their model, consumers use the equilibrium price as a reference point, as we do in this chapter. However, deals are simultaneously evaluated and search costs play no role. In Zhou (2011) the offer of the first firm that consumers visit serves as a reference point. He finds that the more 'prominent' firm whose product is taken as the reference point by more consumers will randomize between a high and a low price. Consumer loss aversion in the price dimension intensifies competition while that in the product dimension softens competition. Contrary to Zhou (2011) we endogenize the search order of the firms. In Karle and Peitz (2014), consumers are located on a Salop circle and know prices before reference points are formed. Spiegler (2012) takes Heidhues and Köszegi (2008) and Köszegi and Rabin (2006) as a starting point. However, in this paper consumers are only loss averse in the price dimension and consumers have a single reference point which equals the expected price of the monopolistic firm. The setup leads to prices being lower with loss aversion. Karle (2015) studies the effect of loss aversion on advertising, while Rosato (2014) considers a monopolist that sells differentiated products and takes advantage of loss aversion to use limited availability to steer consumers towards the more profitable product.

The remainder of this chapter is structured as follows. Section 2.2 introduces the standard model of search with differentiated products, that serves as a benchmark to our model with reference dependent consumers. In Section 2.3 we introduce loss aversion in that model and give the formal requirements for a solution. Existence of equilibrium is considered in Section 2.4. To derive an explicit solution and do comparative statics, we focus on the case of uniformly distributed match values in Section 2.5. Section 2.6 introduces our equilibrium refinement and studies its implications. We consider two extensions in Section 2.7: one with an alternative reference point in the match-value dimension (Section 2.7.1), and one that also allows for gain utility (Section 2.7.2). Section 2.8 concludes.

2.2 Benchmark Model

For ease of exposition, we start with the standard model of search with differentiated products that does not have loss aversion. This largely follows Anderson and Renault (1999). Consider a market where n firms sell horizontally differentiated products. Marginal costs are constant and normalized to zero.Consider one consumer among the infinitely many on the market. Absent loss aversion, she derives utility

$$U^i(p_i, \varepsilon_i) \equiv v + u(p_i, \varepsilon_i) = v + \varepsilon_i - p_i \tag{2.1}$$

if she buys product i at price p_i, $i = 1, \ldots, n$, where the stand alone utility of buyting the product, v, is large enough such that the consumer always buys in equilibrium. We assume that ε_i is the realization of a random variable with distribution F and continuously differentiable log-concave[1] density f with support normalized to $[0, 1]$. It can be interpreted as a match value between the consumer and the good sold by firm i. Match values are independently distributed across products. Firms cannot observe ε_i so price discrimination is not feasible. The consumer only learns ε_i and p_i upon visiting

[1]Log-concavity of f is a common assumption in the literature. The densities of the normal and uniform distribution are for instance log-concave.

firm i. To visit firm i the consumer must incur a search cost s. She searches sequentially with perfect recall.

Suppose the buyer approaches some firm i in her first search. Assume for now that all firms charge the equilibrium price p^*. If the consumer buys from i she obtains gross utility $v + u^i$, with $u^i \equiv u(p^*, \varepsilon_i) = \varepsilon_i - p^*$. A visit to some other firm j will give her utility $v + \varepsilon_j - p^*$. This is higher than the utility from buying from firm i if $\varepsilon_j > \varepsilon_i$. The expected benefit from searching once more then equals

$$b(\varepsilon_i) = \int_{\varepsilon_i}^{1} (\varepsilon - \varepsilon_i) f(\varepsilon) \, d\varepsilon. \tag{2.2}$$

The consumer is willing to do an additional search whenever $b(\varepsilon_i) \geq s$, or $\varepsilon_i \leq \hat{\varepsilon}$, with $\hat{\varepsilon}$ implicitly defined by

$$b(\hat{\varepsilon}) = s. \tag{2.3}$$

To derive p^*, suppose firm i defects by setting some p_i. Define $\Delta \equiv p_i - p^*$. The probability that the consumer stops searching at i conditional on her visiting i, is given by

$$\Pr[\varepsilon_i - p_i > \hat{\varepsilon} - p^*] = \Pr[\varepsilon_i > \hat{\varepsilon} + \Delta] = 1 - F(\hat{\varepsilon} + \Delta). \tag{2.4}$$

The probability that i is sampled first equals $1/n$. Suppose a different firm is sampled first. The probability that the consumer will not buy there equals $F(\hat{\varepsilon})$. Hence, the probability that i is sampled second is $F(\hat{\varepsilon})/n$. More generally, the probability that it is sampled kth is $F(\hat{\varepsilon})^{k-1}/n$, $k = 1, \ldots, n$. The probability that the consumer buys from i is then given by

$$D(p_i, p^*) = \frac{1}{n} \sum_{k=0}^{n-1} F(\hat{\varepsilon})^k \left[1 - F(\hat{\varepsilon} + \Delta)\right] + \int_{-\infty}^{\hat{\varepsilon}+\Delta} F(\varepsilon - \Delta)^{n-1} f(\varepsilon) \, d\varepsilon. \tag{2.5}$$

The last term reflects the probability that the consumer returns to firm i after having visited all firms and learning that i offers the highest net utility. Maximizing profits $\pi_i(p_i, p^*) = p_i D(p_i, p^*)$ with respect to p_i and imposing symmetry, equilibrium prices can be shown to equal

$$p^* = \frac{1}{\frac{1-F(\hat{\varepsilon})^n}{1-F(\hat{\varepsilon})} f(\hat{\varepsilon}) - n \int_{-\infty}^{\hat{\varepsilon}} f'(\varepsilon) F(\varepsilon)^{n-1} d\varepsilon}. \tag{2.6}$$

Anderson and Renault (1999) give conditions under which this is indeed a global maximum.

In the remainder of this chapter we will look at the case of infinitely many firms, so $n \to \infty$. This greatly simplifies the analysis, and allows us to do comparative statics. With infinitely many firms, a consumer will never return to a firm that she has visited before, so the integral in (2.5) vanishes. Taking the limit of (2.6), equilibrium prices then equal

$$p^* = \frac{1 - F(\hat{\varepsilon})}{f(\hat{\varepsilon})}. \tag{2.7}$$

2.3 Search with loss aversion

Introduction We now introduce reference-dependent utility to this model. Following Köszegi and Rabin (2006), we look for a personal equilibrium; a consumption plan consistent with optimal behavior, given expectations, and given a reference point that consumers are using.

Before a consumer embarks on the search for a product, she has expectations concerning the price she will end up paying, the match value she will end up with, and the amount of search cost she has to incur. Hence, in all these three dimensions there is scope for disappointment or pleasant surprises. Consistent with earlier literature, we assume that losses in each dimension are evaluated separately, so that all disappointments and pleasant surprises enter the utility function additively.

A consumer obtains a disutility λx over and above her standard utility, if the outcome is x worse than the reference point that she expected in that dimension. As in standard gain-loss utility theory, we could also allow for an additional utility $\lambda_G y$ when the outcome is y better than her reference point, with $\lambda_G = \gamma\lambda$ and $\gamma < 1$. For simplicity we will not do so (and hence set $\gamma = 0$) in the main text, and consider the case $\gamma > 0$ in Section 2.7.2.

Köszegi and Rabin (2006) explicitly allow decision makers to use reference distributions rather than reference points. For example, if a consumer expects a match value from a distribution G on $[0, 1]$ and she finds some ε, then that is a disappointment relative to all realizations higher than ε, weighted by their likelihood. Hence, in that case she has loss utility $\lambda \int_{\varepsilon}^{1} (\xi - \varepsilon)\, g(\xi) d\xi$.

It is crucial how and when the reference points are determined against which losses are evaluated. In Heidhues and Köszegi (2008), consumers expect to find the equilibrium price p^* and are disappointed when they find a price $p_i > p^*$. Moreover, they expect to experience some match value $\hat{\chi}$, and are disappointed whenever $\chi < \hat{\chi}$. In Karle and Peitz (2014) consumers already know the *distributions* of prices and firm match values before forming reference points: the reference distributions are the distributions of prices and match values. We take a similar approach in our model.[2]

In the *price dimension*, we take the reference price to be the (tentative) equilibrium price p^*. Hence, a consumer is disappointed and experiences additional loss utility if the firm she visits has a price $p_i > p^*$. In the *search dimension*, we take the reference point to be the probability that she continues search. With reservation utility $\hat{\varepsilon}$, she will continue search with probability $F(\hat{\varepsilon})$. If she does continue search, that will be a disappointment relative to that probability.

It is less straightforward to determine the proper reference point in the *match-value dimension.* Suppose a consumer visits a firm and finds match value ε. First, note that she expected to find a draw from F on $[0, 1]$. Hence, we could use that distribution as the reference distribution. Second, when using reservation utility $\hat{\varepsilon}$, this consumer knows in advance that she will continue search until she finds a match value $\varepsilon \geq \hat{\varepsilon}$. Hence, she knows in advance that she will end up with a match value ε that is a draw from F, but conditional on it being bigger than $\hat{\varepsilon}$. Therefore, we could also use that as our match value distribution. Third, if the consumer finds some match value ε_i, but she continues search, one could also argue that her proper reference point should be ε_i, as that is her best match value so far.

The third option would however yield technical problems as with optimal search it would lead to a reservation utility $\hat{\varepsilon}$ that is no longer stationary, as it depends on the match values that have already been sampled. That yields huge technical difficulties, hence we prefer not to take that route. In the remainder of the paper we choose the second option; the consumer expects to find some $\varepsilon \geq \hat{\varepsilon}$, hence any lower ε would be a disappointment. In Section 2.7.1, we consider the first option in an extension.

[2] The use of the term 'match value' is somewhat confusing in this context, as both papers consider a Salop circle rather than a model with match values in the sense of Perloff and Salop (1985), as we do. The crucial difference is that in our case match values are independently distributed across firms. On a Salop circle, whenever a consumer moves closer to a particular firm, her match value with that firm increases, but her match value with the firm located at the other side of her necessarily decreases.

2

Personal equilibrium We look for a reservation utility that is consistent with personal equilibrium. In the discussion that follows, we thus have to distinguish between a possible reservation utility $\hat{\epsilon}$ that a consumer could use, and the reservation utility $\hat{\varepsilon}$ that is consistent with personal equilibrium. We thus look for a particular $\hat{\epsilon}$ such that, when rationally taking all possible disappointments into account a priori, our consumer indeed wants to continue search whenever she finds an $\varepsilon_i < \hat{\epsilon}$. Consistent with our analysis without loss aversion, we denote the $\hat{\epsilon}$ for which that holds as $\hat{\varepsilon}$.

As argued, loss aversion affects a consumer's decision in 3 possible ways; in the match utility dimension, the price dimension, and the search dimension. In the *match dimension*, we define the adjusted match utility as the utility the consumer obtains from consuming the product of firm i, after taking into account any disappointments[3] from that product. Obviously, this will depend on the original match utility ε_i, but also on $\hat{\epsilon}$: a higher $\hat{\epsilon}$ implies that the consumer expects to end up with a higher ε_i and hence will be more disappointed if the match utility she finds falls short of $\hat{\epsilon}$. We will formalize this in more detail below. Hence we can write the adjusted match utility as $\varepsilon_L(\varepsilon_i; \hat{\epsilon})$, where the subscript L denotes that we take loss aversion into account. Absent loss aversion, we simply have $\varepsilon_L(\varepsilon_i, \hat{\epsilon}) = \varepsilon_i$.

Disutility in the *price dimension* will be affected by the price firm i charges, p_i, but also by the price the consumer expects to find – which will be the (tentative) equilibrium price p^*. Hence, for price disutility we can write $p_L(p_i; p^*)$. Absent loss aversion, we simply have $p_L(p_i; p^*) = p_i$. In the *search dimension*, search disutility will depend on the search cost plus the extent of disappointment with the amount of search she undertakes. The latter will be affected by how often she expected to search, which is determined by $\hat{\epsilon}$. Hence, for search disutility we can write $s_L(s, \hat{\epsilon})$. The utility a consumer obtains when she buys at firm i is now given by

$$U_L^i(p_i, \varepsilon_i; p^*, \hat{\epsilon}) = v + \varepsilon_L(\varepsilon_i; \hat{\epsilon}) - p_L(p_i; p^*).$$

At equal prices, we denote the expected benefits from search when observing ε_i and using reservation utility $\hat{\epsilon}$ as $b_L(\varepsilon_i, \hat{\epsilon})$. Along the lines in the previous section, the consumer search rule is consistent with personal equilibrium if, at equal prices, whenever she encounters an $\varepsilon_i = \hat{\epsilon}$, she is indifferent between continuing search and buying. Hence,

[3]Or pleasant surprises, in the case that $\gamma > 0$.

a personal equilibrium has the consumer continue searching when she encounters an $\varepsilon_i < \hat{\varepsilon}$, with $\hat{\varepsilon}$ implicitly defined by

$$b_L(\hat{\varepsilon};\hat{\varepsilon}) = s_L(s;\hat{\varepsilon}).$$

We will now derive the functions we have defined above.

Analysis First, consider the *match dimension.* By construction, the consumer continues search until she finds a match value at least equal to $\hat{\epsilon}$. Hence, she expects to obtain a match value that is a draw from the distribution

$$H(\xi;\hat{\epsilon}) = \Pr(\varepsilon < \xi | \xi > \hat{\epsilon}) = \frac{F(\xi) - F(\hat{\epsilon})}{1 - F(\hat{\epsilon})}. \tag{2.8}$$

Thus, H is the posterior distribution of ε, given that it is higher than $\hat{\epsilon}$. For every value of ε_i that a consumer may encounter at firm i, the *adjusted match utility* ε_L after taking into account any disappointment disutility due to reference dependence, now equals

$$\varepsilon_L(\varepsilon_i;\hat{\epsilon}) = \begin{cases} \varepsilon_i - \lambda_M \int_{\hat{\epsilon}}^{\infty} (\tau - \varepsilon_i)\,\mathrm{d}H(\tau) & \text{if } \varepsilon_i \leq \hat{\epsilon} \\ \varepsilon_i - \lambda_M \int_{\varepsilon_i}^{\infty} (\tau - \varepsilon_i)\mathrm{d}H(\tau) & \text{if } \varepsilon_i \geq \hat{\epsilon}. \end{cases} \tag{2.9}$$

Here, λ_M is the loss aversion parameter in the match value dimension. We believe that the most natural assumption is to have the loss aversion parameter being equal in all three dimensions that we consider. Still, in our analysis we do allow them to differ. This allows us to trace the effects that we find to the three channels we consider. It is easy to see that adjusted match utility is strictly increasing in ε_i. We denote its cumulative probability distribution by $F_L(\varepsilon;\hat{\epsilon})$. The adjusted match utility distribution then follows straightforwardly from the unadjusted match value distribution $F(\varepsilon)$.

In the *price dimension*, the consumer will experience a loss utility whenever $p_i > p^*$. Hence

$$p_L(p_i;p^*) = p_i + \lambda_P(p_i - p^*)\mathbf{1}_{\{p_i > p^*\}},$$

with $\mathbf{1}$ the indicator function and λ_P the loss aversion parameter in the price dimension. In the *search dimension*, note that the consumer expects to buy directly from a firm with probability $1 - F(\hat{\epsilon})$, and to continue search with $F(\hat{\epsilon})$. Hence, if she decides to continue searching, this is a disappointment of size s relative to all realizations of ε where she did not expect to search. The disutility from searching is then given by

$$s_L(s;\hat{\epsilon}) = s + s\lambda_S[1 - F(\hat{\epsilon})], \tag{2.10}$$

with λ_S the loss utility parameter in the search dimension. For the expected benefit of search we immediately have

$$b_L(\varepsilon_i;\hat{\epsilon}) = \int_{\varepsilon_i}^{\infty} \left[\varepsilon_L\left(\varepsilon;\hat{\epsilon}\right) - \varepsilon_L\left(\varepsilon_i;\hat{\epsilon}\right)\right] dF\left(\varepsilon\right). \tag{2.11}$$

A consumer thus continues search if $\varepsilon^i \leq \hat{\varepsilon}$, with $\hat{\varepsilon}$ implicitly defined by[4]

$$b_L(\hat{\varepsilon};\hat{\varepsilon}) = s + \lambda_S s\left[1 - F\left(\hat{\varepsilon}\right)\right]. \tag{2.12}$$

Equilibrium Derivation of the price equilibrium p^* goes along the same lines as above. Define $\hat{\varepsilon}_L \equiv \varepsilon_L\left(\hat{\varepsilon};\hat{\varepsilon}\right)$. Hence, $\hat{\varepsilon}_L$ is the adjusted match utility that makes the consumer indifferent between buying immediately and continuing search, if prices are equal. Suppose all other firms set price p^*, but firm i charges some price p_i. We now have to distinguish between two cases: one in which $p_i > p^*$ (and the consumer is disappointed when observing p_i), and one in which $p_i \leq p^*$ (and that is not the case). We first restrict attention to downward defections, so $p_i < p^*$. Denote the tentative equilibrium price as p_A^* and define $\Delta_A \equiv p_i - p_A^*$. Similar to Section 2.2, the consumer stops searching whenever $\varepsilon_L^i - p_i \geq \hat{\varepsilon}_L - p_A^*$ or $\varepsilon_L^i \geq \hat{\varepsilon}_L + \Delta_A$. The probability that she stops at i given that i is visited, is equal to

$$\Pr[\varepsilon_L\left(\varepsilon_i;\hat{\varepsilon}\right) > \hat{\varepsilon}_L + \Delta_A] = 1 - F_L(\hat{\varepsilon}_L + \Delta_A;\hat{\varepsilon}). \tag{2.13}$$

This implies that demand for firm i in the case of n firms is

$$\begin{aligned} D_i(p_i, p_A^*) &= \frac{1}{n}[1 - F_L(\hat{\varepsilon}_L + \Delta_A;\hat{\varepsilon})]\frac{1 - F_L(\hat{\varepsilon}_L;\hat{\varepsilon})^n}{1 - F_L(\hat{\varepsilon}_L;\hat{\varepsilon})} \\ &+ \int_{-\infty}^{\hat{\varepsilon}_L+\Delta_A} F_L(\varepsilon_L - \Delta_A;\hat{\varepsilon})^{n-1}\mathrm{d}F_L(\varepsilon_L;\hat{\varepsilon}). \end{aligned} \tag{2.14}$$

Profits for this case are $\Pi(p_i;p_A^*) = p_i D_i(p_i, p_A^*)$. Taking the first order condition, imposing symmetry, and taking the limit for $n \to \infty$, we have

$$p_A^* = \frac{-D_i(p_A^*, p_A^*)}{\frac{\partial D_i(p_A^*, p_A^*)}{\partial p_i}} = \frac{1 - F_L(\hat{\varepsilon}_L;\hat{\varepsilon})}{f_L(\hat{\varepsilon}_L;\hat{\varepsilon})}, \tag{2.15}$$

[4]Note that $b_L(\hat{\epsilon};\hat{\epsilon})$ is decreasing in $\hat{\epsilon}$. We assume that s is small enough such that $b_L(0,0) > s+s\lambda_S$; otherwise there would never be any search. Also, $b_L(\infty,\infty) = 0$. The term, $s + \lambda_S S\left(1 - F(\hat{\epsilon})\right))$ is decreasing in $\hat{\epsilon}$; from $s + s\lambda_S$ if $\hat{\epsilon} = 0$ to s if $\hat{\epsilon} \to \infty$. Hence, by continuity a solution always exists (but might not be unique).

provided the maximization problem is well defined and such a p_A^* exists.

Now consider upward defections, with $p_i > p^*$. Denote the tentative equilibrium price as p_B^* and define $\Delta_B \equiv p_i - p_B^*$. The consumer will stop searching at i whenever $\varepsilon_L^i - p_i - \lambda_P \Delta_B \geq \hat{\varepsilon}_L - p_B^*$ or $\varepsilon_L^i \geq \hat{\varepsilon}_L + (1 + \lambda_P)\, \Delta_B$. The probability that she stops at i given that i is visited, is equal to

$$\Pr[\varepsilon_L\,(\varepsilon_i) > \hat{\varepsilon}_L + (1+\lambda_P)\,\Delta_B] = 1 - F_L(\hat{\varepsilon}_L + (1+\lambda_P)\,\Delta_B). \tag{2.16}$$

Maximizing and imposing symmetry yields

$$p_B^* = \frac{1 - F_L(\hat{\varepsilon}_L; \hat{\varepsilon})}{(1+\lambda_P)\, f_L(\hat{\varepsilon}_L; \hat{\varepsilon})} \tag{2.17}$$

again provided the maximization problem is well defined and such a p_B^* exists. We thus have $p_B^* = p_A^* / \,(1+\lambda_P)$, hence $p_B^* \leq p_A^*$.

Hence, when allowing for downward defections, we find that p_A^* is the unique equilibrium price. This implies that firms do have an incentive to defect to a lower price from any tentative equilibrium $p^* > p_A^*$, but not from a $p^* < p_A^*$. When allowing for upward defections, we find that p_B^* is the unique equilibrium price. This implies that firms do have an incentive to defect to a higher price from any tentative equilibrium $p^* < p_B^*$, but not from a $p^* > p_B^*$. Given that $p_B^* \leq p_A^*$, this implies that firms have no incentive to defect from any $p^* \in [p_B^*, p_A^*]$. We thus have a continuum of Nash equilibria.

For the general case, it is hard to do comparative statics. In Section 2.5 we therefore simplify by assuming that $F(\varepsilon)$ is a uniform distribution.[5] But we first discuss conditions for the existence of equilibrium in the general case.

2.4 Existence of equilibrium

We showed that our search model with loss aversion has a continuum of equilibria $[p_B^*, p_A^*]$, provided that those values exist. From Anderson and Renault (1999), absent loss aversion the equilibrium exists if $1 - F$ is logconcave and the number of firms goes to infinity (their Proposition B1). With a finite number of firms, it is sufficient to have $1 - F$ logconcave and $f' \geq 0$ (which follows directly from their Proposition B2). In our

[5] If $n < \infty$ a consumer no longer is certain she will end up with a match value of at least $\hat{\varepsilon}$. This fact complicates H and F_L and hence the analysis severely. Moreover, consumers might return after visiting all n firms to an earlier visited firm. We thus have to evaluate $\int_{-\infty}^{\hat{\varepsilon}_L + \Delta_B[1+\lambda_P I_{\{\Delta_B>0\}}]} F_L\left(\varepsilon_L - \Delta_B[1+\lambda_P I_{\{\Delta_B>0\}}]\right)^{n-1} \mathrm{d}F_L\left(\varepsilon_L\right)$, which is non-trivial.

2

case, as we focus on a monopolistic setting, this immediately implies that p_B^* and p_A^* exist if $1-F_L$ is logconcave. In this section, we show that logconcavity of $1-F$ and $f' \geq 0$ is sufficient for that to hold.

We closely follow Theorem 7 in Bagnoli and Bergstrom (2005), that looks at logconcavity of F rather than $1-F$, in a similar problem. First note $(\ln(1-F))' = -f/(1-F)$, so

$$(\ln(1-F))'' = \frac{-(1-F)f' - f^2}{(1-F)^2}.$$

Logconcavity of $1-F$ thus implies that

$$\frac{f'}{f} + \frac{f}{1-F} > 0. \tag{2.18}$$

Consider $F_L \equiv F(\varepsilon_L(\varepsilon))$. We have $(\ln(1-F(\varepsilon_L(\varepsilon))))' = -f\varepsilon_L'/(1-F)$ so

$$(\ln(1-F(\varepsilon_L(\varepsilon))))'' = \frac{-(1-F)\left(f'\cdot(\varepsilon_L')^2 + f\cdot\varepsilon_L''\right) - (f\cdot\varepsilon_L')^2}{(1-F)^2}.$$

Hence, logconcavity of $1-F_L$ requires[6]

$$\frac{f'}{f} + \frac{\varepsilon_L''}{(\varepsilon_L')^2} + \frac{f}{1-F} > 0. \tag{2.19}$$

From (2.9), we have

$$\varepsilon_L' = \begin{cases} 1+\lambda_M \int_{\hat{\varepsilon}}^{\infty} \mathrm{d}H(\tau) & \text{if } \varepsilon \leq \hat{\varepsilon} \\ 1+\lambda_M \int_{\varepsilon}^{\infty} \mathrm{d}H(\tau) & \text{if } \varepsilon > \hat{\varepsilon}, \end{cases}$$

and

$$\varepsilon_L'' = \begin{cases} 0 & \text{if } \varepsilon \leq \hat{\varepsilon} \\ -\lambda_M h(\varepsilon) & \text{if } \varepsilon > \hat{\varepsilon}. \end{cases}$$

Therefore, for $\varepsilon \leq \hat{\varepsilon}$, condition (2.19) is always satisfied if (2.18) is. Let's now focus on the case $\varepsilon > \hat{\varepsilon}$. Note from (2.8) that

$$\begin{aligned} h(\varepsilon) &= \frac{f(\varepsilon)}{1-F(\hat{\varepsilon})}, \\ \int_{\varepsilon}^{\infty} \mathrm{d}H(\tau) &= 1-H(\varepsilon) = \frac{1-F(\varepsilon)}{1-F(\hat{\varepsilon})}. \end{aligned}$$

We thus have

$$\frac{\varepsilon_L''}{(\varepsilon_L')^2} = \frac{-\lambda_M f(\varepsilon)}{(1-F(\hat{\varepsilon}))\left(1+\lambda_M\left(\frac{1-F(\varepsilon)}{1-F(\hat{\varepsilon})}\right)\right)^2}.$$

[6] The square is missing in Bagnoli and Bergstrom (2005). We take that to be a typo.

Write $\text{Den} \equiv 1 - F(\hat{\varepsilon})$. In that case

$$\begin{aligned}
\frac{\varepsilon_L''}{(\varepsilon_L')^2} + \frac{f}{1-F} &= \frac{f}{1-F} - \frac{\lambda f}{\text{Den}\left(1+\lambda\left(\frac{1-F}{\text{Den}}\right)\right)^2} \\
&= f \cdot \frac{\lambda^2 (1-F)^2 + \lambda \text{Den}(1-F) + \text{Den}^2}{(1-F)(\lambda(1-F)+\text{Den})^2} > 0.
\end{aligned}$$

That implies that $f' \geq 0$ is sufficient for (2.19) to be satisfied.

2

2.5 A uniform distribution of match values

In this section, we consider a uniform distribution of match values on $[0,1]$. Note that this domain is just for ease of exposition; a wider range of match values, for example, is equivalent to having lower search costs.

Distribution of ε_L Given that a consumer continues search until she finds $\varepsilon_i \geq \hat{\epsilon}$, the match utility she ends up with (not taking into account the effects of loss aversion) is uniformly distributed on $[\hat{\epsilon}, 1]$. Hence, following (2.8),

$$H(\xi; \hat{\epsilon}) = \frac{\xi - \hat{\epsilon}}{1 - \hat{\epsilon}},$$

so $h(\xi, \hat{\epsilon}) = 1/(1-\hat{\epsilon})$. From (2.9), we then have

$$\varepsilon_L(\varepsilon_i; \hat{\epsilon}) = \begin{cases} \varepsilon_i - \frac{1}{2}\lambda_M (1 + \hat{\epsilon} - 2\varepsilon_i) & \text{if } \varepsilon_i \leq \hat{\epsilon} \\ \varepsilon_i - \frac{1}{2}\lambda_M \cdot \frac{(1-\varepsilon_i)^2}{1-\hat{\epsilon}} & \text{if } \varepsilon_i \geq \hat{\epsilon}. \end{cases} \tag{2.20}$$

Note that $\varepsilon_L(\hat{\epsilon}; \hat{\epsilon}) = \lim_{\varepsilon_i \uparrow \hat{\epsilon}} \varepsilon_L(\varepsilon_i; \hat{\epsilon}) = \lim_{\varepsilon_i \downarrow \hat{\epsilon}} \varepsilon_L(\varepsilon_i; \hat{\epsilon}) = \hat{\epsilon} - \frac{1}{2}\lambda_M(1-\hat{\epsilon})$. First consider the case that $\varepsilon_i \leq \hat{\epsilon}$ or $\varepsilon_L \leq \hat{\epsilon} - \frac{1}{2}\lambda_M(1-\hat{\epsilon})$. Then

$$\begin{aligned}
F_L(\xi; \hat{\epsilon}) &= \Pr(\varepsilon_L \leq \xi) = \Pr\left(\varepsilon_i - \frac{1}{2}\lambda_M (1 + \hat{\epsilon} - 2\varepsilon_i) \leq \xi\right) \\
&= \Pr\left(\varepsilon_i \leq \frac{\xi + \frac{1}{2}\lambda_M (1+\hat{\epsilon})}{1+\lambda_M}\right) = \frac{\xi + \frac{1}{2}\lambda_M (1+\hat{\epsilon})}{1+\lambda_M}.
\end{aligned} \tag{2.21}$$

With $\varepsilon_i \geq \hat{\epsilon}$ we have

$$\begin{aligned}
F_L(\xi; \hat{\epsilon}) &= \Pr\left(\varepsilon_i - \lambda_M \frac{(1-\varepsilon_i)^2}{2(1-\hat{\epsilon})} \leq \xi\right) \\
&= \Pr\left(\varepsilon_i \leq 1 + \frac{1-\hat{\epsilon}}{\lambda_M}\left(1 - \sqrt{1 + \frac{2\lambda_M (1-\xi)}{1-\hat{\epsilon}}}\right)\right)
\end{aligned}$$

$$= \quad 1+\frac{1-\hat{\epsilon}}{\lambda_M}\left(1-\sqrt{1+\frac{2\lambda_M\,(1-\xi)}{1-\hat{\epsilon}}}\right). \tag{2.22}$$

From (2.21) and (2.22),

$$\lim_{\xi\uparrow\hat{\epsilon}} f_L(\xi;\hat{\epsilon}) = \lim_{\xi\downarrow\hat{\epsilon}} f_L(\xi;\hat{\epsilon}) = \frac{1}{1+\lambda_M}.$$

2

Equilibrium prices We first determine $\hat{\varepsilon}$. From (2.11), we have

$$\begin{aligned} b_L(\hat{\epsilon};\hat{\epsilon}) &= \int_{\hat{\epsilon}}^{1}\left(\varepsilon-\frac{1}{2}\lambda_M\left(\frac{(1-\varepsilon)^2}{1-\hat{\epsilon}}\right)-\left(\hat{\epsilon}-\frac{1}{2}\lambda_M\,(1-\hat{\epsilon})\right)\right)\mathrm{d}\varepsilon \\ &= \frac{1}{6}\,(1-\hat{\epsilon})^2\,(2\lambda_M+3)\,. \end{aligned} \tag{2.23}$$

Equating this to $s_L\,(s;\hat{\epsilon}) \equiv s\,(1+\lambda_S\,(1-\hat{\epsilon}))$ yields

$$\hat{\varepsilon} = 1-\frac{3s\lambda_S}{2\lambda_M+3}-\frac{1}{2\lambda_M+3}\sqrt{9s^2\lambda_S^2+12s\lambda_M+18s}, \tag{2.24}$$

as the other root is larger than 1.[7] With $\hat{\varepsilon}_L \leq \hat{\varepsilon}$, we have $f_L(\hat{\varepsilon}_L) = 1/\,(1+\lambda_M)$. From (2.15) and (2.17) we then have:

Proposition 2.1. *When match values are uniformly distributed on* $[0,1]$, *there is a continuum of Nash equilibria on* $[p_B^*, p_A^*]$, *with*

$$\begin{aligned} p_A^* &= (1-\hat{\varepsilon})\,(1+\lambda_M) \\ &= \frac{1+\lambda_M}{2\lambda_M+3}\left(3s\lambda_S+\sqrt{9s^2\lambda_S^2+12s\lambda_M+18s}\right), \end{aligned} \tag{2.25}$$

$$p_B^* = \frac{(1-\hat{\varepsilon})\,(1+\lambda_M)}{(1+\lambda_P)} = \frac{p_A^*}{1+\lambda_p}. \tag{2.26}$$

From Section 2.4, sufficient for existence is for $1-F$ to be logconcave, and to have $f' \geq 0$. Both conditions are clearly satisfied.

The impact of loss aversion We can now show the following:

Proposition 2.2. *The effect of loss aversion on the upper bound on equilibrium prices* p_A^* *and the lower bound* p_B^*, *is as follows:*

[7] Absent loss aversion, we have $(1-\hat{\varepsilon})^2/2 = s$, so for the model to make sense, we need $s \leq 1/2$, otherwise a consumer would never search beyond the first firm. With loss aversion

$$\frac{(2\lambda+3)(1-\hat{\varepsilon})^2}{6\,(1+\lambda(1-\hat{\varepsilon}))} = s.$$

The left-hand side is highest for $\hat{\varepsilon}=0$, when it becomes $\frac{(2\lambda+3)}{6(1+\lambda)} = s$. Note that $\lim_{\lambda\to\infty}\frac{(2\lambda+3)}{6(1+\lambda)} = \frac{1}{3}$. Hence, we're fine as long as $s \leq 1/3$.

a. *An increase in loss aversion λ increases p_A^*; it increases p_B^* when search costs are sufficiently high, but decreases it otherwise.*

b. *Loss aversion in the search and match dimensions increase p_A^*, while loss aversion in the price dimension has no effect on p_A^*.*

c. *Loss aversion in the search and match dimension increase p_B^*, but loss aversion in the price dimension decreases p_B^*.*

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Proof. For the impact of λ_M, note from (2.25) that the first two terms are strictly increasing in λ_M. This implies that p_A^* also is. The same holds for p_B^* in (2.26). The effects of λ_S and λ_P follow immediately from (2.25) and (2.26).

For a common λ, equilibrium prices collapse to

$$p_A^* = \frac{1+\lambda}{2\lambda+3}\left(3s\lambda+\sqrt{9s\lambda^2+12s\lambda+18s}\right) \quad (2.27)$$

$$p_B^* = \frac{1}{2\lambda+3}\left(3s\lambda+\sqrt{9s\lambda^2+12s\lambda+18s}\right). \quad (2.28)$$

With $\frac{\partial p_A^*}{\partial \lambda_S}, \frac{\partial p_A^*}{\partial \lambda_M} > 0$ and $\frac{\partial p_A^*}{\partial \lambda_p} = 0$, we immediately have $\frac{\partial p_A^*}{\partial \lambda} > 0$. The impact on p_B^* is more involved. Defining $R \equiv \sqrt{9s\lambda^2+12s\lambda+18s}$, we can write

$$\frac{\partial p_B^*}{\partial \lambda} = \frac{\left(\frac{9s^2\lambda+6s}{R}+3s\right)(2\lambda+3)-2(3s\lambda+R)}{(2\lambda+3)^2}$$

This has the same sign as

$$\begin{aligned}
& \left(9s^2\lambda+6s+3sR\right)(2\lambda+3)-2(3s\lambda+R)R \\
= \; & -2R^2+9Rs+3s(2\lambda+3)(3s\lambda+2) && (2.29)\\
= \; & -2\left(9s^2\lambda^2+12s\lambda+18s\right)+9Rs+3s(2\lambda+3)(3s\lambda+2) \\
= \; & 9Rs-3s(6+4\lambda-9s\lambda). && (2.30)
\end{aligned}$$

With $s < 1/3$, we have $6+4\lambda-9s\lambda > 0$. Hence, (2.30) is positive if and only if $3R > 6+4\lambda-9s\lambda$ or

$$9s\left(9\lambda^2+12\lambda+18\right)-(6+4\lambda-9s\lambda)^2 > 0.$$

For $s=0$, the expression is $-(6+4\lambda)^2 < 0$, For $s=1/3$, it equals $26\lambda^2+24\lambda+18 > 0$. Also, the expression is strictly increasing in s.[8] Hence, there is a unique $\hat{s} \in (0,1/3)$

[8]The derivative with respect to s equals $9\left(9\lambda^2+12\lambda+18\right)+18\lambda(6+4\lambda-9s\lambda) = 216\lambda+153\lambda^2-162s\lambda^2+162 \geq 216\lambda+153\lambda^2-162\lambda^2/3+162 = 99\lambda^2+216\lambda+162 > 0$.

such that $\frac{\partial p_B^*}{\partial \lambda} \leq 0$ for $s < \hat{s}$ and $\frac{\partial p_B^*}{\partial \lambda} > 0$ otherwise.[9] ■

We thus find that loss aversion leads to a range of possible Nash equilibria, as is also the case in e.g. Heidhues and Köszegi (2008). Yet, different from their work, we find that although the upper bound on equilibrium prices increases in the extent of loss aversion, the lower bound might decrease. Hence, loss aversion may lead to lower prices.

Loss aversion in the search dimension leads to unambiguously higher effective search costs, as having to continue search will always entail some disappointment. This unambiguously leads to higher equilibrium prices. Loss aversion in the price dimension implies that consumers are more likely to walk away when they see a price increase, which leads to weakly lower prices.[10] Loss aversion in the match-value dimension essentially leads to more product differentiation and the ex post distribution of prices will increase;[11] the highest possible match value will yield the same utility level as before, but the lowest possible match value will yield much lower utility, as encountering that will be a huge disappointment. In these models, more product differentiation implies higher prices.[12]

It is easy to see that prices increase in search costs, as they do in the model without loss aversion. With higher search costs, firms have more market power as consumers are less likely to walk away.

The amount of search It is also interesting to consider the effect of loss aversion on the equilibrium amount of search. We have the following:

Proposition 2.3. *More loss aversion leads to more search when search costs are low, but to less search when search costs are high. More loss aversion in the match-value dimension leads to more search, more loss aversion in the search dimension leads to*

[9] Solving explicitly for $\hat{s}$ gives a particularly ugly expression that is strictly decreasing in λ and equals $\hat{s} = 2/9$ for $\lambda = 0$ and $\hat{s} = 1/9$ for $\lambda \to \infty$.

[10] In the sense that the lower bound decreases. The upper bound is unaffected; by construction that is the equilibrium price when only considering *downward* defections.

[11] The adjusted match value has a support on $[-\frac{1}{2}\lambda_M(1+\hat{\varepsilon}), 1]$. The original match value had a support on $[0, 1]$. This means that products are more differentiated if we are able to show that the density under loss aversion is at every point of the original support smaller than before. The density in case of no loss aversion is constant at 1, while it now equals $1/\sqrt{1+\frac{2\lambda_M(1-\varepsilon_L)}{1-\hat{\varepsilon}}}$. Since $\varepsilon_L \in [0, 1]$ on this original support and $\hat{\varepsilon} \in [0, 1]$ the result follows.

[12] See e.g. Proposition 2 in Anderson and Renault (1999).

less search, while loss aversion in the price dimension has no effect on the amount of search.

Proof. From (2.26), when loss aversion is equal in all dimensions, we have $p_B^* = 1 - \hat{\varepsilon}$. Above we showed $\partial p_B^*/\partial\lambda \leq 0$ if and only if $s < \hat{s}$. That immediately implies that $\partial\hat{\varepsilon}/\partial\lambda \geq 0$, so there will be more search, if and only if $s < \hat{s}$. Now consider loss aversion in the match-value dimension. Suppose $\varepsilon \geq \hat{\varepsilon}$. From (2.9), we then have

$$\varepsilon_L(\varepsilon;\hat{\varepsilon}) - \varepsilon_L(\hat{\varepsilon};\hat{\varepsilon}) \geq \varepsilon_i - \hat{\varepsilon}$$

for any $\hat{\varepsilon}$. It then immediately follows that

$$b_L(\hat{\varepsilon},\hat{\varepsilon}) = \int_{\hat{\varepsilon}}^{1} \left(\varepsilon_L(\varepsilon;\hat{\varepsilon}) - \varepsilon_L(\hat{\varepsilon};\hat{\varepsilon})\right) \mathrm{d}F(\varepsilon) \geq \int_{\hat{\varepsilon}}^{1} (\varepsilon - \hat{\varepsilon}) \,\mathrm{d}F(\varepsilon) = b(\hat{\varepsilon})$$

for any $\hat{\varepsilon}$. This implies the following. Suppose that $\hat{\varepsilon}$ is the consumer's reservation utility in the model with no loss aversion, so $b(\hat{\varepsilon}) = s$. Then, using that same $\hat{\varepsilon}$, the consumer with loss aversion will always have a strictly larger benefit from search. It follows that loss aversion leads to more search on the market. The result on the search dimension follows by taking the derivative of (2.24) with respect to λ_S. As we can see in (2.24), loss aversion in the price dimension, λ_P, has no effect on $\hat{\varepsilon}$. ■

2.6 Equilibrium Refinement

In the previous sections, we solved the model to find a continuum of possible Nash equilibria. In this section, we therefore propose an equilibrium refinement that yields a unique equilibrium. The idea is as follows. In principle, any equilibrium $p^* \in [p_B^*, p_A^*]$ could be played. Hence, consumers and firms have to coordinate on one of these infinitely many equilibria. Suppose that we allow consumers to make small mistakes in the equilibrium price that is to be played. More precisely, suppose than any given consumer expects $p^* + \mu$ to be played, rather than the 'true' equilibrium price p^*, with μ a draw from a uniform distribution on $[-\bar{\mu}, \bar{\mu}]$.[13] Under this assumption, we can again

[13] We implicitly assume that consumers do not alter their beliefs even after observing the true equilibrium price at several firms. One alternative is to assume that if a consumer visited at least two firms with the true equilibrium price p^* she alters her expectations to $p^e = p^*$. However, this would imply that a consumer may return to the first firm after having visited the second, as her expectation with respect to the prices that she will find at the remaining shops has now changed. This would complicate the analysis tremendously.

derive all equilibrium prices. If taking the limit of that sequence of equilibrium prices when $\bar{\mu} \to 0$ yields a unique equilibrium price, then that is the price we expect to be played.[14]

We can now show the following:

Proposition 2.4. *The unique price that survives our equilibrium refinement is given by*

$$\bar{p}^* = \frac{1 - F_L\left(\hat{\varepsilon}_L\right)}{f_L(\hat{\varepsilon}_L)\left(1 + \lambda/2\right)}.$$

Proof. We first derive demand of firm i when it sets price p_i while all others set p^*, and consumers make the mistakes mentioned above. Consider a consumer who visits firm i. Suppose she expects some $p^e = p^* + \mu$, with $\mu < 0$. Naturally, as she still expects all prices to be equal, $\hat{\varepsilon}_L$ will not be affected. Then, for p_i sufficiently close to p^*, she will buy with probability

$$\begin{aligned}
&\Pr(\varepsilon_i - p_i - \lambda_P(p_i - p^* - \mu) > \hat{\varepsilon}_L - p^* - \mu) = \\
&\Pr(\varepsilon_i - \lambda_P(\Delta - \mu) > \hat{\varepsilon}_L + \Delta - \mu) = 1 - F_L(\hat{\varepsilon}_L + (1 + \lambda_P)\left(\Delta - \mu\right)).
\end{aligned}$$

A consumer who visits firm i and expects some $p^e = p^* + \mu$ with $\mu > 0$ will buy with probability

$$\Pr(\varepsilon_i > \hat{\varepsilon}_L + \Delta - \mu) = 1 - F_L(\hat{\varepsilon}_L + \Delta - \mu).$$

Total demand for firm i in this refinement, if i charges price p_i, where the equilibrium price is p^* and the upper bound on consumer mistakes is $\bar{\mu}$, is now given by

$$\begin{aligned}
\bar{D}_i\left(p_i, p^*; \bar{\mu}\right) &= \int_{-\bar{\mu}}^{0} \frac{1 - F_L(\hat{\varepsilon}_L + (1 + \lambda_P)\left(\Delta - \mu\right))}{2\bar{\mu}} \mathrm{d}\mu + \int_{0}^{\bar{\mu}} \frac{1 - F_L(\hat{\varepsilon}_L + \Delta - \mu)}{2\bar{\mu}} \mathrm{d}\mu \\
&= 1 - \int_{0}^{\bar{\mu}} \frac{F_L(\hat{\varepsilon}_L + (1 + \lambda_P)\left(\Delta + \mu\right)) + F_L(\hat{\varepsilon}_L + \Delta - \mu)}{2\bar{\mu}} \mathrm{d}\mu.
\end{aligned}$$

Note that we no longer have to distinguish between profits when doing an upward defection, and profits when doing a downward defection, as regardless of the direction of the defection, there are always consumers who expected a higher price and those

[14] As an alternative interpretation, our refinement is also consistent with a world in which consumers do not make these mistakes, but firms perceive them to do so.

that expected a lower price. Profits now equal $\bar{\Pi}(p_i, p^*; \bar{\mu}) = p_i \bar{D}_i(p_i, p^*; \bar{\mu})$, so the equilibrium price is

$$p^*(\bar{\mu}) = \frac{D_i(p^*, p^*; \bar{\mu})}{-\frac{\partial D_i(p^*, p^*; \bar{\mu})}{\partial p_i}},$$

while the unique equilibrium that survives this refinement is given by $\bar{p}^* = \lim_{\bar{\mu} \to 0} p^*(\bar{\mu})$. Note that

$$\frac{\partial D_i(p_i, p^*; \bar{\mu})}{\partial p_i} = -\int_0^{\bar{\mu}} \frac{f_L(\hat{\varepsilon}_L + (1+\lambda_P)(\Delta + \mu))(1+\lambda_P) + f_L(\hat{\varepsilon}_L + \Delta - \mu)}{2\bar{\mu}} \mathrm{d}\mu.$$

Evaluated at $\Delta = 0$, this yields

$$\frac{\partial D_i(p^*, p^*; \bar{\mu})}{\partial p_i} = -\int_0^{\bar{\mu}} \frac{f_L(\hat{\varepsilon}_L + (1+\lambda_P)\mu)(1+\lambda_P) + f_L(\hat{\varepsilon}_L - \mu)}{2\bar{\mu}} \mathrm{d}\mu,$$

so, using de L'Hôpital,

$$\lim_{\bar{\mu} \to 0} \frac{\partial D_i(p^*, p^*; \bar{\mu})}{\partial p_i} = -f_L(\hat{\varepsilon}_L)(1 + \lambda_P/2),$$

which implies the result. ■

In case of a uniform distribution, we have from the analysis in Section 2.5,

$$\bar{p}^* = \frac{1+\lambda_M}{(2\lambda_M + 3)(1 + \lambda_P/2)} \left(3s\lambda_S + \sqrt{9s^2\lambda_S^2 + 12\lambda_M s + 18s}\right).$$

This is increasing in λ_M, decreasing in λ_P and increasing in λ_S. With the same λ in all dimensions, the expression collapses to

$$\bar{p}^* = \frac{2(1+\lambda)}{(2\lambda + 3)(2 + \lambda)} \left(3s\lambda + \sqrt{9s^2\lambda^2 + 12\lambda s + 18s}\right).$$

Unfortunately, it is hard to do comparative statics for this expression. A numerical analysis shows that for large s, it is always increasing in λ, while for small s it is increasing in λ for small enough λ, but decreasing for larger λ. For example, Figure 2.1 gives $\bar{p}^*$ as a function of λ for a number of values of s.

2.7 Extensions

2.7.1 An alternative reference point

We now consider the case where a consumer uses the distribution of match values at firm i as a reference point, rather than the distribution of reference points that she may

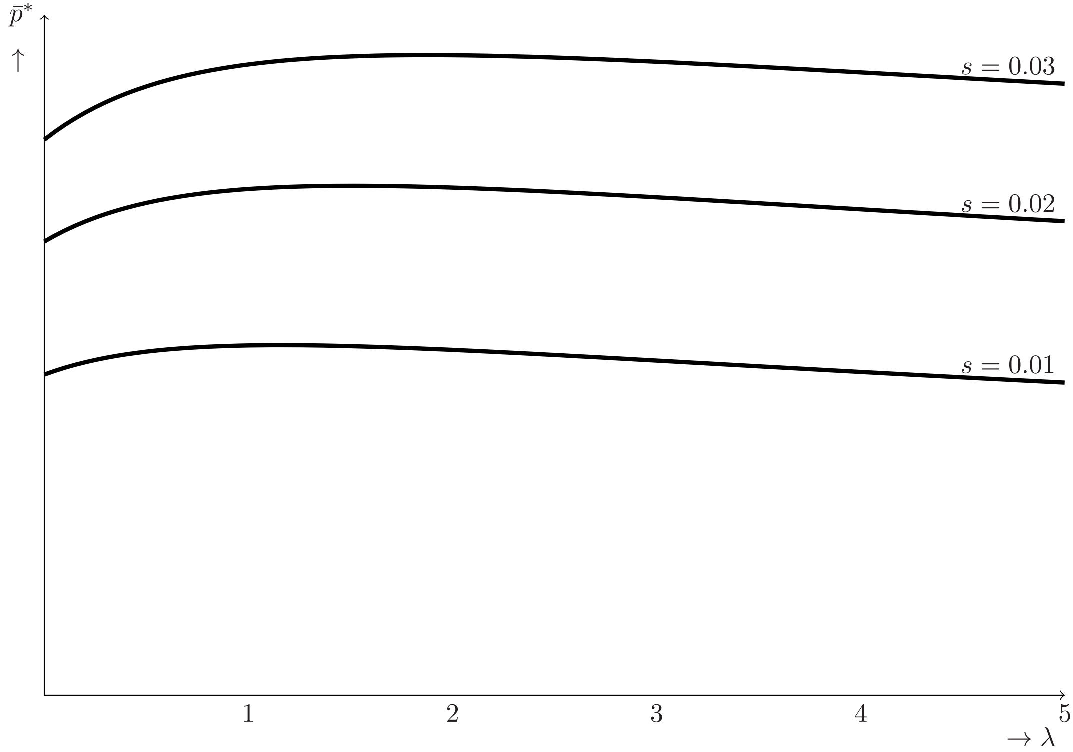

Figure 2.1: $\bar{p}^*$ as a function of λ, for assorted values of s.

end up with. The adjusted match utility then equals

$$\varepsilon_L(\varepsilon_i) = \varepsilon_i - \lambda_M \int_{\epsilon_i}^{1} (\tau - \varepsilon_i) \mathrm{d}F(\tau),$$

as this is a disappointment relative to the higher values of ε_i that could have been achieved. As the consumer no longer takes into account the posterior distribution that she will end up with, the adjusted match value no longer depend on the reservation utility $\hat{\epsilon}$, which greatly simplifies the analysis. We denote the distribution of adjusted match values as $G(\varepsilon)$.

Utility in the search and price dimensions are not affected relative to the analysis in our main text. Hence, we still have

$$s_L\left(s; \hat{\epsilon}\right) = s + \lambda_S s \left(1 - F\left(\hat{\epsilon}\right)\right)$$

and

$$p_L\left(p_i; p^*\right) = p_i + \lambda_P \left(p_i - p^*\right) \mathbf{1}_{\{p_i > p^*\}},$$

while the benefits of search also no longer depend on $\hat{\epsilon}$;

$$b_L(\varepsilon_i) = \int_{\varepsilon_i}^{\infty} \left(\varepsilon_L\left(\varepsilon\right) - \varepsilon_L\left(\varepsilon_i\right)\right) dF\left(\varepsilon\right).$$

The reservation utility $\hat{\varepsilon}$ is now implicitly defined by

$$b_L(\hat{\varepsilon}) = s + \lambda_S s \left(1 - F\left(\hat{\varepsilon}\right)\right).$$

Derivation of the price equilibrium p^* goes along the same lines as in the main analysis. Define $\hat{\varepsilon}_L \equiv \varepsilon_L\left(\hat{\varepsilon}\right)$. Suppose that all other firms set price p^*, but i charges some price p_i. First consider downward defections from tentative equilibrium price p_A^*, with $\Delta_A \equiv p_i - p_A^*$. Again, the consumer will stop searching whenever $\varepsilon_L^i \geq \hat{\varepsilon}_L + \Delta_A$. With n firms, demand for i then is

$$D_i(p_i; p_A^*) = \frac{1}{n}[1 - G(\hat{\varepsilon}_L + \Delta_A)]\frac{1 - G(\hat{\varepsilon}_L)^n}{1 - G(\hat{\varepsilon}_L)} + \int_{-\infty}^{\hat{\varepsilon}_L + \Delta_A} G(\varepsilon - \Delta_A)^{n-1} g(\varepsilon) \mathrm{d}\varepsilon.$$

Maximizing profits $\Pi(p_i; p_A^*) = p_i D_i(p_i, p_A^*)$ with respect to p_i and imposing symmetry, we have

$$p_A^* = \frac{1}{g(\hat{\varepsilon}_L)\frac{1-G(\hat{\varepsilon}_L)^n}{1-G(\hat{\varepsilon}_L)} - n\int_{-\infty}^{\hat{\varepsilon}_L} g'(\varepsilon)G(\varepsilon)^{n-1}\mathrm{d}\varepsilon}.$$

Now consider an upward deviation p_i from tentative equilibrium price p_B^*, with $\Delta_B \equiv p_i - p_B^*$. The consumer will now stop searching at firm i whenever $\varepsilon_L^i - p_i - \lambda_P \Delta_B \geq \hat{\varepsilon}_L - p_B^*$ or $\varepsilon_L^i \geq \hat{\varepsilon}_L + \left(1 + \lambda_P\right)\Delta_B$. This yields demand

$$\begin{aligned} D_i(p_i; p_B^*) &= \frac{1}{n}[1 - G(\hat{\varepsilon}_L + \left(1 + \lambda_P\right)\Delta_B)]\frac{1 - G(\hat{\varepsilon}_L)^n}{1 - G(\hat{\varepsilon}_L)} \\ &+ \int_{-\infty}^{\hat{\varepsilon}_L + (1+\lambda_P)\Delta_B} G(\varepsilon - \left(1 + \lambda_P\right)\Delta_B)^{n-1} g(\varepsilon)\mathrm{d}\varepsilon. \end{aligned}$$

This yields

$$p_B^* = \frac{1}{\left(1 + \lambda_P\right)\left[\frac{1-G(\hat{\varepsilon}_L)^n}{1-G(\hat{\varepsilon}_L)} g(\hat{\varepsilon}_L) - n\int_{-\infty}^{\hat{\varepsilon}_L} g'\left(\varepsilon\right) G\left(\varepsilon\right)^{n-1} d\varepsilon\right]}.$$

Also in this case, we thus have $p_B^* = p_A^* / \left(1 + \lambda_P\right)$, and a continuum of Nash equilibria on $\left[p_B^*, p_A^*\right]$.

Comparative statics For comparative statics, we again assume infinitely many firms and a uniform distribution of match values. Then

$$\varepsilon_L(\varepsilon) = \varepsilon - \lambda_M \int_{\varepsilon}^{1} (\tau - \varepsilon) \mathrm{d}\tau = \varepsilon - \frac{1}{2}\lambda_M (1-\varepsilon)^2 .$$

The cutoff value $\hat{\varepsilon}$ is thus the solution to

$$\int_{\hat{\varepsilon}}^{1} \left(\varepsilon - \frac{1}{2}\lambda_M (1-\varepsilon)^2 + \hat{\varepsilon} - \frac{1}{2}\lambda_M (1-\hat{\varepsilon})^2 \right) d\varepsilon = s + \lambda_S s (1-\hat{\varepsilon}) . \tag{2.31}$$

In this case, it is rather impractical to find an explicit solution to $\hat{\varepsilon}$, as it involves solving a polynomial of the third-degree. With equal λ's, we can show however that $\hat{\varepsilon}$ exists and is strictly between 0 and 1.

Existence of $\hat{\varepsilon}$ Let's look at the case of equal λ. Define

$$W(x) \equiv \frac{1}{6}(1-x)^2(2\lambda(1-x)+3) - s - \lambda(1-x)s. \tag{2.32}$$

Hence (2.31) is satisfied if $\hat{\varepsilon}$ is a root of $W(x)$. With $s < 1/3$, we have $W(0) = \frac{1}{6}(2\lambda + 3) - s(1+\lambda) > 0$. Also, $W(1) = -s < 0$. Continuity then implies $\exists x$ in $[0,1]$ such that $W(x) = 0$. As $W''(x) = 2\lambda(1-x) + 1$, W is strictly convex on $[0,1]$. This implies that $W(x) = 0$ has a unique solution on $[0,1]$.

The amount of search Taking the total differential of $W(\hat{\varepsilon}) = 0$, we have

$$\frac{d\hat{\varepsilon}}{d\lambda} = -\frac{\partial W/\partial\lambda}{\partial W/\partial\hat{\varepsilon}}.$$

Convexity of $W(x)$ implies that it has a unique minimum $\tilde{x}$ on $[0,1]$. Necessarily, $\hat{\varepsilon} \in [0,\tilde{x}]$ and $W(x) < 0$ for $x \in [\hat{\varepsilon}, 1]$. Moreover $W(1-\sqrt{3s}) = \frac{1}{2}s > 0$, hence $\hat{\varepsilon} > 1 - \sqrt{3s}$. Since $\hat{\varepsilon} > 1 - \sqrt{3s}$ we have $\frac{\partial W}{\partial \lambda}(\hat{\varepsilon}) = \frac{1}{3}(1-\varepsilon)^3 - (1-\varepsilon)s < 0$ and $\partial W/\partial\hat{\varepsilon} = -\lambda(1-x)^2 - (1-x) + s\lambda < 0$. This implies that $d\hat{\varepsilon}/d\lambda < 0$, so more loss aversion here leads to more search.

Equilibrium To solve the firms' problem, first note

$$\begin{aligned} G(\varepsilon_L) &= \Pr(\varepsilon_L(\varepsilon_i) < \varepsilon_L) = \Pr\left(\varepsilon_i - \frac{1}{2}\lambda_M(1-\varepsilon_i)^2 < \varepsilon_L\right) \\ &= 1 + \frac{1}{\lambda_M}\left(1 - \sqrt{1 + 2\lambda_M(1-\varepsilon_L)}\right). \end{aligned}$$

Note that the lower bound of the support of G is given by $\varepsilon_L(0) = -\frac{1}{2}\lambda_M$, while the upper bound is $\varepsilon_L(1) = 1$. We thus have $\varepsilon_L \in \left[-\frac{1}{2}\lambda_M, 0\right]$. The density g is given by

$$g(\varepsilon_L) = \frac{1}{\sqrt{1 + 2\lambda_M(1 - \varepsilon_L)}}.$$

Moreover,

$$g'(\varepsilon_L) = \frac{\lambda_M}{(1 + 2\lambda_M(1 - \varepsilon_L))^{3/2}} > 0.$$

We now have:

Proposition 2.5. *When consumers use the entire distribution $F(\varepsilon)$ as their reference distribution in the match-value dimension, there is a continuum of Nash equilibria on $[p_B^*, p_A^*]$, with*

$$\begin{aligned} p_A^* &= \frac{1 - G(\hat{\varepsilon}_L)}{g(\hat{\varepsilon}_L)} = \frac{1}{\lambda_M}\left(2\lambda_M(1 - \hat{\varepsilon}_L) + 1 - \sqrt{1 + 2\lambda_M(1 - \hat{\varepsilon}_L)}\right), \\ p_B^* &= \frac{p_A^*}{1 + \lambda_P} = \frac{1}{\lambda_M(1 + \lambda_P)}\left(2\lambda_M(1 - \hat{\varepsilon}_L) + 1 - \sqrt{1 + 2\lambda_M(1 - \hat{\varepsilon}_L)}\right), \end{aligned}$$

with $\hat{\varepsilon}_L = \hat{\varepsilon} - \frac{1}{2}\lambda_M(1 - \hat{\varepsilon})^2$, and $\hat{\varepsilon}$ implicitly defined by (2.31). The effect of loss aversion on p_A^* and p_B^* is as follows:

a. An increase in loss aversion λ increases p_A^*, while the effect on p_B^* is ambiguous.

b. Loss aversion in the search and match dimensions increase p_A^*, while loss aversion in the price dimension has no effect on p_A^*.

c. Loss aversion in the search and match dimensions increase p_B^*, but loss aversion in the price dimension decreases p_B^*.

Proof. First consider $\partial\hat{\varepsilon}_L/\partial\lambda$. Define

$$Q(x) = \int_{G(x)}^{1} (\varepsilon - \frac{1}{2}\lambda(1 - \varepsilon)^2 - x)\mathrm{d}\varepsilon - s - s\lambda(1 - \hat{\varepsilon}).$$

Note that the integral in $Q(x)$ has a lower bound that is increasing in λ while the integrand is decreasing in λ. It is immediate that the function $\varepsilon_L(x)$ is decreasing in λ. Furthermore, since $\partial\hat{\varepsilon}/\partial\lambda \leq 0$ we have:

$$\frac{\partial Q(x)}{\partial\lambda} = \frac{\partial\varepsilon_L}{\partial\lambda} - s(1 - \hat{\varepsilon}) + s\lambda\frac{\partial\hat{\varepsilon}}{\partial\lambda} \leq 0. \tag{2.33}$$

Hence, Q shifts downwards when λ increases, implying $\frac{\partial \hat{\varepsilon}_L}{\partial \lambda} \leq 0$. Now consider

$$\frac{\partial p_A^*}{\partial \lambda} = \frac{-2\lambda^2 \frac{\partial \hat{\varepsilon}_L}{\partial \lambda} - \lambda \frac{(1-\hat{\varepsilon}_L)-\lambda\frac{\partial \hat{\varepsilon}_L}{\partial \lambda}}{\sqrt{2\lambda(1-\hat{\varepsilon}_L)+1}} - 1 + \sqrt{2\lambda(1-\hat{\varepsilon}_L)+1}}{\lambda^2}.$$

The numerator has the same sign as:

$$\frac{\partial \hat{\varepsilon}_L}{\partial \lambda}\left[-2\lambda^2\sqrt{1+2\lambda(1-\hat{\varepsilon}_L)}+\lambda^2\right] + \lambda(1-\hat{\varepsilon}_L)+1-\sqrt{1+2\lambda(1-\hat{\varepsilon}_L)}.$$

Since $-2\lambda^2\sqrt{1+2\lambda(1-\hat{\varepsilon}_L)}+\lambda^2 \leq 0$ the first term is positive. The sum of the other terms also is. Hence $\frac{\partial p_A^*}{\partial \lambda} \geq 0$. For the effect on p_B^*, note that

$$\frac{\partial p_B^*}{\partial \lambda} = \frac{(1+\lambda)\frac{\partial p_A^*}{\partial \lambda} - p_A^*}{(1+\lambda)^2}.$$

The sign of the numerator is ambiguous.

Loss aversion in the search dimension leads to higher effective search costs, which leads to higher equilibrium prices. It is immediate that p_A^* decreases in λ_P but there is no effect on p_B^*. Finally,

$$\frac{\partial p_A^*}{\partial \lambda_M} = \frac{1}{\lambda_M^2\sqrt{2\lambda_M(1-\hat{\varepsilon}_L)+1}} \times$$
$$\left(\lambda_M(1-\hat{\varepsilon}_L)+1-\frac{\partial \hat{\varepsilon}_L}{\partial \lambda_M}\left(2\lambda_M^2\sqrt{2\lambda_M(1-\hat{\varepsilon}_L)+1}-\lambda_M^2\right)-\sqrt{2\lambda_M(1-\hat{\varepsilon}_L)+1}\right).$$

This expression is clearly positive since $\lambda_M(1-\hat{\varepsilon}_L)+1 \geq \sqrt{1+2\lambda_M(1-\hat{\varepsilon}_L)}$ and $\frac{\partial \hat{\varepsilon}_L}{\partial \lambda_M} \leq 0$. Therefore $\frac{\partial p_A^*}{\partial \lambda_M} \geq 0$. The result for p_B^* follows directly. ■

Hence, the effect of loss aversion is comparable to that in the model in the main text; the higher bound on prices increases while the effect on the lower bound is ambiguous. The other comparative statics are also qualitatively the same as in our main analysis. Also, since $\hat{\varepsilon}_L$ is decreasing in s, we immediately have that p_A^* and p_B^* are increasing in s.

Finally, we consider the unique equilibrium price that satisfies our equilibrium refinement in this context. We now have:

Proposition 2.6. *The unique equilibrium price that survives our equilibrium refinement is decreasing in λ.*

Proof. Along the exact same lines as in the main analysis, we have

$$\bar{p}^* = \frac{1}{\lambda\left(1+\lambda/2\right)}\left(2\lambda(1-\hat{\varepsilon}_L)+1-\sqrt{1+2\lambda(1-\hat{\varepsilon}_L)}\right)$$

Denoting $K \equiv 1-\hat{\varepsilon}$,

$$\frac{\partial\bar{p}^*}{\partial\lambda} = \frac{\lambda\left(1+\lambda/2\right)\left(2K-\frac{K}{\sqrt{2K\lambda+1}}\right)-\left(2\lambda K+1-\sqrt{1+2\lambda K}\right)\left(1+\lambda\right)}{\lambda^2\left(1+\lambda/2\right)^2}.$$

The numerator can be written

$$\frac{1}{2}\frac{2K\lambda+3K\lambda^2+2+2\lambda-2\sqrt{2K\lambda+1}\left(\lambda+K\lambda^2+1\right)}{\sqrt{2K\lambda+1}}.$$

Hence, sufficient for this to be negative is that $2K\lambda+3K\lambda^2+2+2\lambda < 2\sqrt{2K\lambda+1}\left(\lambda+K\lambda^2+1\right)$ or

$$\left(2K\lambda+3K\lambda^2+2+2\lambda\right)^2 < 4\left(2K\lambda+1\right)\left(\lambda+K\lambda^2+1\right)^2$$

which implies $-K\lambda^2\left(8K^2\lambda^3+11K\lambda^2+4K\lambda+4\lambda+4\left(1-K\right)\right) < 0$ which is always satisfied. ■

2.7.2 Gain utility

In this section, we consider the case that $\gamma > 0$, so our consumer also experiences gain utility. In line with psychological evidence, we assume $\gamma < 1$.

In the match dimension, the posterior distribution of match values is again given by (2.8). A loss averse consumer that obtains a value ε at a firm will obtain adjusted match utility

$$\varepsilon_L(\varepsilon_i;\hat{\epsilon}) = \begin{cases} \varepsilon_i - \lambda_M\int_{\hat{\epsilon}}^{\infty}\left(\tau-\varepsilon_i\right)\mathrm{d}H\left(\tau\right) & \text{if } \varepsilon_i \leq \hat{\epsilon} \\ \varepsilon_i - \lambda_M\int_{\varepsilon_i}^{\infty}(\tau-\varepsilon_i)\mathrm{d}H(\tau)+\gamma\lambda_M\int_{\hat{\epsilon}}^{\varepsilon_i}(\varepsilon_i-\tau)\mathrm{d}H(\tau) & \text{if } \varepsilon_i \geq \hat{\epsilon} \end{cases} \tag{2.34}$$

as there are only pleasant surprises if $\varepsilon_i > \hat{\epsilon}$. We denote the resulting distribution F_γ.

In the price dimension, we now have

$$p_L\left(p_i;p^*\right) = p_i + \lambda_P\left(p_i-p^*\right)\mathbf{1}_{\{p_i>p^*\}} - \gamma\lambda_P\left(p^*-p_i\right)\mathbf{1}_{\{p_i<p^*\}}.$$

In the search dimension, again, with probability $1-F(\hat{\varepsilon})$, the consumer expects not to continue search, hence it is a disappointment if she does decide to search. However, with probability $F(\hat{\varepsilon})$ she does expect to search, and it is a pleasant surprise if she does not have to. But that implies that her utility, if she would not search, would now equal $\gamma\lambda_S sF(\hat{\varepsilon})$. Hence, the opportunity costs of continuing search equal

$$s_L = s + \lambda_S s\left(1 - F\left(\hat{\varepsilon}\right)\right) + \gamma\lambda_S sF\left(\hat{\varepsilon}\right). \tag{2.35}$$

Surprisingly, gain utility does also act to increase effective search costs.

With these adjustments, a personal equilibrium is given by

$$b_L(\hat{\varepsilon};\hat{\varepsilon}) = s_L + \lambda_S s_L\left[1 - F\left(\hat{\varepsilon}\right)\right],$$

where $b_L\left(\varepsilon_i, \hat{\epsilon}\right)$ is again given by (2.11). Using the same analysis as in the main text, there is now a continuum $[p_B^*, p_A^*]$ of Nash equilibria, with

$$p_A^* = \frac{1 - F_\gamma(\hat{\varepsilon}_L)}{(1+\gamma\lambda_P)f_\gamma(\hat{\varepsilon}_L)} \tag{2.36}$$

and

$$p_B^* = \frac{p_A^*(1+\gamma\lambda_P)}{1+\lambda_P}. \tag{2.37}$$

With gain utility, the range of equilibria is thus smaller; the range collapses to a unique equilibrium if $\gamma \to 1$.[15] The analysis concerning existence is very similar to the case of $\gamma = 0$; also here, $f' \geq 0$ and logconcavity of $1 - F$ are sufficient for these equilibria to exist.

It is hard to find an explicit expression for the range of equilibrium prices, even in the case of a uniform distribution of match values; solving for $\hat{\epsilon}$ then involves solving a third-degree polynomial. However, do note from the above that an increase in γ in the search dimension leads to higher effective search costs, and hence to higher prices. An increase in γ in the match dimension implies that the range of adjusted match values becomes larger compared to a case with no gain utility, but with loss aversion. This yields unambiguously higher prices. Finally, from (2.36) and (2.37) we immediately have that an increase in γ in the price dimension leads to lower prices. Hence, the net effect is ambiguous.

2.8 Conclusion

We considered a model in which consumers are loss averse and incur search cost to find out the deals on offer by a firm on a market with differentiated products. Loss aversion

[15] In fact, it can be shown that in that case, the unique equilibrium coincides with that in the standard model without gain-loss utility.

then affects consumers through three channels: price, search costs, and match values. In the price dimension, consumers are disappointed when they encounter a price that is higher than expected. This leads to weakly lower prices. In the search dimension, consumers may be disappointed when they have to search more often than expected. This leads to unambiguously higher prices since it effectively increases search costs. In the match-value dimension, consumers are disappointed if they do not like the product as much as they could have. This leads to higher prices as, effectively, products become more differentiated. Loss aversion yields more search when search costs are relatively low, but less search when they are relatively high.

Taken together, the effect of loss aversion on prices is ambiguous. As is usual in these models, we found a range of Nash equilibria. We proposed an equilibrium refinement that effectively assumes that consumers make mistakes that are infinitesimally small. The unique price that survives this refinement is decreasing in the extent of loss aversion for intermediate search costs and relatively high loss aversion, and increasing otherwise. In our base analysis we assumed that consumers take into account that in equilibrium they end up with a match value higher than some lower bound. Any lower match value will then be a disappointment. In an extension we studied the case where the entire distribution of match values acts as a reference. We then found qualitatively similar results.

Chapter 3

Directed Consumer Search*

3.1 Introduction

On many markets there are multiple suppliers who offer products which differ in several dimensions and, in order to find the best offer, consumers will have to search. Consumers can visits seller's shops and websites randomly but nowadays, more often than not, they rely on intermediating parties such as search engines and online marketplaces to obtain advise on which seller is most likely to offer a product and deal to their liking. For instance, if one wants to buy a new DVD, shopping websites suggest titles based on the order history. When people are using an online search engine or comparison website to find a product, they might enter not only the product name but also specify some characteristics they like. It is clear that the internet offers the possibility to let the search order depend upon one's preferences. However, even in the absence of the world-wide-web this has been and still is possible: a person looking to purchase a car with high MPG will not start shopping at a car-dealer who mainly advertises SUV's. Similarly, an employee of a bookstore will not suggest to start looking in the comics section when a consumer indicates that she likes classical literature.

This chapter provides several new insights when one does allows the listing order on intermediating platforms, and therefore the search process of consumers, to depend on consumer preferences. To model the role of the preferences in the listing and search order, I assume that a product consists of so-called communicable and incommunicable

*This chapter is based on Siekman (2015).

horizontal characteristics. Consider, for instance, a consumer wanting to rent a house. In order to find a suitable property she visits a platform on which she might indicate that she has preferences for a house in close proximity to her her new workplace, this is a communicable characteristics. Based upon this information the platform suggest the real-estate agent at which the consumer is most likely to find the house that fits her tastes the best. That is, the consumer is *directed* in her search for the best property. The consumer in turn visits the website of suggested real-estate agent in order to find out the price and inspect the product's incommunicable characteristics, such as whether the floorplan is convenient. She can then rent the house or decide to continue her search until she does find an offer to her liking.

For simplicity this chapter assumes that only the first suggestion by the intermediating platform depends upon the consumer's preferences. This means that if the consumer decides to continue her search, she will visit a random supplier's website. I expect, however, that my findings carry over to the more general framework the entire ranking depends upon consumer characteristics. A justification for only allowing the first suggestion to depend on the consumer's tastes might be that this is profit maximizing for the platform as this place becomes more valuable for a seller as it decreases the expected value of continuing search for a consumer. I however do not model this side of the market.

I find that prices and profits are higher in case of directed instead of random search. This is because sellers exploit the fact that consumers first visit the seller with the highest communicable attributes and are therefore less likely to visit a competitor. In effect, products are ex-ante less homogeneous for consumers when product characteristics are communicated.

This result is different than the one presented in Armstrong et al. (2009), which considers a model similar model to mine, however, there a consumer's preferences does not affect which seller is visited first. In that model one seller is prominent and it is sampled first by every consumer. I do not impose this a-symmetry on sellers. In Armstrong et al. (2009) the prominent seller sets a lower price than in the case of random search as it's demand has become more elastic. The other sellers, on the other hand, face less elastic demand and set a price above the one charged in the case of random search.

This chapter provides some interesting insights on the welfare effects of directed search as well. When product information affects the search order consumers obtain on average a better match, as they start searching at the seller where they are most likely to find the product that fits their tastes the best. Moreover, consumers are less likely to continue search in each stage of the process. This is because they know that for certain characteristics the product of the first seller is the best on the market. Hence, consumers spend less on search costs. These two effects ensure that total welfare is higher under directed search compared to random search. However, consumers turn out to be worse off as these two effects are outweighed by the higher prices they pay.

Again the difference in results with Armstrong et al. (2009) is worth noting. They also find that consumers search less, although for a different reason. In their model a consumer is more likely to buy at the first seller because it sets a lower price than the competition. This induces some consumers not to search beyond the first seller although it would be socially efficient. This means that on average consumers obtain a lower match value than in case of random search. Hence, opposite to me they find that non-random search leads to lower total welfare.

This observation leads to a conclusion that is opposite to that in Armstrong et al. (2009) as well. Suppose there is a profit maximizing platform that provides a link between consumers and sellers. If the platform is able to extract total welfare by charging both consumers and sellers, it will have an incentive for different groups of consumers to suggest different sellers based upon the consumer's preferences.

My model extends the seminal work of Anderson and Renault (1999). In that article consumers visit sellers in a random order, by introducing communicable characteristics I allow consumer preferences to influence the search order. When, in my framework, a product only consists of incommunicable characteristics consumers search at random and the model of Anderson and Renault (1999) results. However, when all product characteristics are used to determine which seller is visited first the Diamond (1971) paradox emerges. The result in Anderson and Renault (1999) that prices are non-decreasing in search costs carries over to my work.

My findings contribute to the literature on consumer search with differentiated products, which is built around the classic papers by Wolinsky (1986) and Anderson and Renault (1999). These and subsequent works often assume totally random search by consumers amongst sellers. However, besides the already discussed Armstrong et al.

(2009), there are some notable exceptions. Arbatskaya (2007) considers a market with homogeneous products where consumers face heterogeneous search costs and search in an exogenously given order. Sellers have knowledge about this search order, and they charge higher prices when they are visited in an early stage. In the work of Zhou (2011) consumers also search in a fixed order, however, products are horizontally differentiated. He finds that prices increase in the search order, since sellers visited in a later stage exploit the fact that consumers who sample them must have relatively low valuations for the offered product of earlier sampled sellers. Other literature in which the search order is non-random includes Wilson (2010), who considers a homogeneous good market in which seller's search costs are endogenized. Sellers can obfuscate, which makes it more time-consuming for consumers to inspect a product and learn its price. In equilibrium Wilson finds that consumers are more likely to visit a seller with low search costs. A similar approach is taken by Ellison and Wolitzky (2013), but they assume consumers can not observe the level of obfuscation prior to arrival. In the empirical work of Hortaçsu and Syverson (2004) the sampling probability of a seller is proxied by advertising expenditures. The work by Haan and Moraga-González (2011) takes a different approach. They consider a consumer search model in which consumers first visit the seller whose advertising is most salient. Finally Haan et al. (2015) consider a duopoly in a similar setting as my framework. However, they focus on the case that information on prices are disclosed. Hence, while there model is more relevant in modeling how advertisements affects the search order, mine is more suitable to analyze the effect of permutations amongst the results of a search engine.

My work connects to the literature on position auctions as well. Influential works in this fields are fore instance Athey and Ellison (2011) and Chen and He (2011). In their models sellers differ in the likelihood that they can meet the 'need' that a consumer has. These models analyze how the platform can maximize profits by auctioning prominent positions to sellers but abstract away from the relation between consumers and sellers.

The remainder of this chapter is organized as follows. Section 3.2 introduces the model. In section 3.3 equilibrium prices are derived. Section 3.4 presents the benchmark model. Comparative statics are discussed in section 3.5. In section 3.6 I conduct a welfare-analysis. Section 3.7 concludes.

3.2 The model

I consider a market with $n \geq 2$ sellers selling horizontally differentiated products. Sellers face constant marginal costs, which I normalize to zero. Demand is assumed to be inelastic. Without loss of generality the market size is normalized to 1. A consumer buying from seller $i \in \{1, \ldots, n\}$ receives utility

$$u_i = (1 - \lambda)\varepsilon_i + \lambda v_i - p_i,$$

with $\lambda \in [0.1]$. Here p_i is the price charged. $(1-\lambda)\varepsilon_i + \lambda v_i$ is the stochastic match value between a consumer and product i. Match values are independently distributed across products, moreover, I assume ε_i and v_i are independently distributed random variables. ε_i is the realization of a distribution F and is referred to as the Incommunicable Part of the Match Value, *IPMV* henceforth. v_i is the realization of a distribution H and is referred to as the Communicable Part of the Match Value, *CPMV* from now on. The total stochastic match value, denoted by $t_i = (1 - \lambda)\varepsilon_i + \lambda v_i$, has a distribution which is the weighted convolution of F and H, which is denoted by M. Let f, h and m be the densities associated with F, H and M, respectively. f and h are taken to be continuous. $[a_F, b_F]$ and $[a_H, b_H]$ respectively denote the supports of F and H on the extended real line. Let the bounds on t_i be denoted by

$$a = (1 - \lambda)a_F + \lambda a_H \quad \text{and} \quad b = (1 - \lambda)b_F + \lambda b_H.$$

Sellers can not discriminate in prices as they are unable to observe match values. In order to find the best combination of match value and price amongst the sellers consumers visit a platform such as a search engine or a price comparison website. On that platform links to sellers are presented, and the consumer can sequaentially search amongst the sellers with perfect recall and agains search costs s. The links on the platform may be ordered in two ways. First, sellers may presented in a random order which does not convey any information to the consumer. Consumers will then randomly search amongst the sellers, this is the benchmark. Second, the seller with highest OPMV for a consumer may be listed first and all other sellers are listed below it in a random order. Such an order conveys some match information which consumers can use in their search process. I will show that it is actually optimal for consumers to first visit the first listed

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seller. This ordening will be refered to as *directed search*, and below the search process is described in more detail.

Stage 1 Consumers arrive at the platform and communicate some of their preferences. One might imagine consumers looking to rent a house. Consumers might have strong preferences for a certain location because of the proximity to their workplace. A consumer indicates to the platform that she is looking for a house at a particular location, this is the communicable characteristic of the product. The platform then presents a list of sellers, with at the top the seller with a house at or closest to the desired location. Hence, the CPMV is the distance to the desired location.

Stage 2 Subsequently the consumer visits the website of the suggested seller and learns about the price and the total match value, inlcuding the IPMV. The consumer takes this into account when calculating the expected benefit of continuing search, which she compares to the costs of doing so. A high price and/or a low IPMV at the first seller might give a strong reason to continue the search for a better deal. If the consumer's search costs are higher than the expected benefit of search she will buy at the current supplier, otherwise the consumer continues her search by visiting another website listed at the platform.[2] In calculating the expected benefit of searching, the consumer incorporates the fact that for any other product the CPMV is smaller than at the first seller. The IPMV in the example where the consumer is looking for a house might be the floorplan of the house. If a consumer dislikes the house's layout she might still want to look at other options on the market, although she knows that these are further away from the desired location.

Notice that the directed search rule is a generalization of the one presented in the standard consumer search models, see for instance Anderson and Renault (1999). In those models, consumers search completely at random without using any match information. The assumption of not using this information is relaxed in our model by

[2]Note that ordening at the platform beyond the first seller is random. Future research might look into the possibility of allowing this order to depend on the CPMV as well. In such a model the consumer faces, in each stage, a different expected benefit of continuing search, depending on the CPMV of the next best seller. This complicates the model severely.

allowing consumers to determine the first seller they visit based on the suggestion of an intermediating platform

It is important to realize that for a certain consumer each seller has a probility of $1/n$ of being listed first, depending on the realization of the IPMV. This in contrast to the models in Armstrong et al. (2009) and Rhodes (2011) where one seller is always prominent for all consumers for some exogenous reason.

3.3 The pricing Nash equilibrium

In this section I derive a symmetric Nash equilibrium in prices. As I look for a symmetric Nash equilibrium, I need to consider the best response of seller $i \in \{1, \ldots, n\}$ when all other sellers set some price p^*. Setting p^* should be optimal for seller i as well. Below I derive the demand and profit function for seller i.

Consider a consumer before entering the market. She gets informed about which seller has $v_m = \max_{j\in\{1,\ldots,n\}}\{v_j\}$, the highest CPMV, and she visits this seller first. Suppose a consumer has decided to visit seller i first: $v_m = v_i$. If she decides to buy from seller i she will receive an utility of $(1-\lambda)\varepsilon_i + \lambda v_i - p_i$. If she continues search and buys from seller j she receives $(1-\lambda)\varepsilon_j + \lambda v_j - p^*$. Define $\Delta = p_i - p^*$ and $x \equiv (1-\lambda)\varepsilon_i + \lambda v_i - \Delta$. When the consumer arrives at seller i she learns x. The consumer is better off at seller j when $t_j > x$. The expected benefit of continuing searching (net of search costs) from seller i is thus given by:

$$g(x, v_m) = E(t_j - x|v_j \leq v_m) = \int\limits_{-\infty}^{v_m} \int\limits_{\frac{x-\lambda v_j}{1-\lambda}}^{\infty} ((1-\lambda)\varepsilon_j + \lambda v_j - x) f(\varepsilon_j) \mathrm{d}\varepsilon_j \frac{h(v_j)}{H(v^m)} \mathrm{d}v_j.$$

Notice that the consumer does take into account that she has already visited the seller with CPMV v_m and therefore the CPMV's for the remaining seller will be lower. This is why the conditional density $h(v_j)/H(v_m)$ figures in this expression.

$g(x, v_m)$ is strictly decreasing in x and goes from $+\infty$ to zero as x goes from $-\infty$ to $+\infty$. Let $\hat{x}$ be implicitly defined by $g(\hat{x}, v_m) = s$, which exists and is unique for each v_m given the above and $s \in (0, \infty)$. It follows that for a consumer it is beneficial to continue search if the total match value at a seller is lower than $\hat{x}$. Notice that $\hat{x}$ depends on v_m, moreover, the random search model à la Anderson and Renault (1999) is obtained when v_m is replaced by b_H in $g(x, v_m)$ or when $\lambda = 0$. In addition, as $s > 0$, $\hat{x} < (1-\lambda)b_F + \lambda v_m$.

I now derive demand for seller i. As I am considering a symmetric Nash equilibrium, the consumer expects that every seller is charging price p^*, and does not anticipate seller i charging p_i. Suppose $v_i = v_m$, so a consumer arrives at seller i first. This consumer does not continue her search and buys from i whenever $(1-\lambda)\varepsilon_i + \lambda v_m > \hat{x}$. This happens, for a given v_m, with probability $1 - F\left(\frac{\hat{x}+\Delta-\lambda v_m}{1-\lambda}\right)$. The probability that the seller i has the highest CPMV on the market is $H(v_m)^{n-1}$. Therefore, the fraction of the population arriving at i first and staying there to buy can be found by multiplying these two terms, weighing this term with density of v_m, and then integrating over v_m. This procedure yields

$$Q_1(p_i, p^*) = \int_{-\infty}^{\infty} H(v_m)^{n-1} \left[1 - F\left(\frac{\hat{x}+\Delta-\lambda v_m}{1-\lambda}\right)\right] h(v_m)\mathrm{d}v_m. \tag{3.1}$$

Now suppose seller i is visited as l^{th} seller, $l \in \{2, \ldots, n\}$. Let the subscripts of the CPMV and IPMV denote the order in which sellers are visited. In this scenario the consumer must have visited first a seller with CPMV v_m and she must have rejected the deal offered there. For a given v_m this happens with probability $F\left(\frac{\hat{x}-\lambda v_m}{1-\lambda}\right)$. Subsequently the consumer must have visited sellers 2 up till and including $l-1$ and also rejected their offers, implying that $(1-\lambda)\varepsilon_j + \lambda v_j < \hat{x}$ for $j \in \{2, \ldots, l-1\}$. The probability that the consumer continues search from seller j, for a given v_m, equals

$$R(\hat{x}, v_m) = \int_{-\infty}^{v_m} F\left(\frac{\hat{x}-\lambda v_j}{1-\lambda}\right) h(v_j)\mathrm{d}v_j,$$

When the consumer arrives after $l-1$ sellers at seller i she should buy there. Hence, we need $(1-\lambda)\varepsilon_i + \lambda v_i - p_i > \hat{x} - p^*$ and similar as above this happens, for a given v_m, with probability

$$\int_{-\infty}^{v_m} \left[1 - F\left(\frac{\hat{x}+\Delta-\lambda v_i}{1-\lambda}\right)\right] h(v_i)\mathrm{d}v_i.$$

These probabilities already impose that $v_j < v_m$ for $j \in \{2, \ldots, l-1\}$.and $v_i < v_m$. However, we also need that $v_j < v_m$ for the sellers the consumer does not visit, which is captured by the factor $H(v_m)^{n-l}$. By taking these factors into account, weighing them with the density of v_m, and by integrating over v_m one finds that the probability that a consumer buys from seller i upon arrival, given that i is sampled as l^{th} seller, equals

$$\begin{aligned} Q_l(p_i,p^*) &= \int_{-\infty}^{\infty}\int_{-\infty}^{v_m} F\left(\frac{\hat{x}-\lambda v_m}{1-\lambda}\right) R(\hat{x},v_m)^{l-2} \\ &\quad \cdot \left[1-F\left(\frac{\hat{x}+\Delta-\lambda v_i}{1-\lambda}\right)\right] H(v_m)^{n-l} h(v_i)\mathrm{d}v_i h(v_m)\mathrm{d}v_m, \end{aligned} \tag{3.2}$$

In addition there are consumers who buy from seller i after they initially rejected its offer and visited all sellers. The term *comebacks* will be used for these consumers. First consider consumers who visited seller i first ($v_i = \max_{j\in\{1,\ldots,n\}}\{v_j\}$) and return there in the end. The probability that seller i offers a better deal than seller j, for given CPMV's, equals $F\left(\frac{(1-\lambda)\varepsilon_i+\lambda v_m-\Delta-\lambda v_j}{1-\lambda}\right)$. Therefore, we find that demand for seller i from comebacks who visited i first equals

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$$Y^1(p_i,p^*) = \int_{-\infty}^{\infty}\int_{-\infty}^{\frac{\hat{x}+\Delta-\lambda v_m}{1-\lambda}} \left(\int_{-\infty}^{v_m} F\left(\frac{(1-\lambda)\varepsilon_i+\lambda v_m-\Delta-\lambda v_j}{1-\lambda}\right) h(v_j)\mathrm{d}v_j\right)^{n-1} f(\varepsilon_i)\mathrm{d}\varepsilon_i h(v_m)\mathrm{d}v_m,$$

where the upperbound on the second integral ensures that initially the consumer continued search from seller i. Now consider consumers who visit i as seller $l \in \{2,\ldots,n\}$, so $v_i < v_m$, and buys there after considering all the other offers on the market. Similarly as above, we find that this happens with probability:

$$\begin{aligned} Y^2(p_i,p^*) = (n-1)\int_{-\infty}^{\infty}\int_{-\infty}^{v_m}\int_{-\infty}^{\frac{\hat{x}+\Delta-\lambda v_i}{1-\lambda}} &\left[\int_{-\infty}^{v_m} F\left(\frac{(1-\lambda)\varepsilon_i+\lambda v_i-\Delta-\lambda v_j}{1-\lambda}\right) h(v_j)\mathrm{d}v_j\right]^{n-2} \\ &\cdot F\left(\frac{(1-\lambda)\varepsilon_i+\lambda v_i-\Delta-\lambda v_m}{1-\lambda}\right) f(\varepsilon_i)\mathrm{d}\varepsilon_i h(v_i)\mathrm{d}v_i h(v_m)\mathrm{d}v_m. \end{aligned}$$

It follows that total demand for seller i charging p_i, given all other sellers charge p^*, is:

$$D^D(p_i,p^*) = Q_1(p_i,p^*) + \sum_{l=2}^{n} Q_l(p_i,p^*) + Y^1(p_i,p^*) + Y^2(p_i,p^*). \tag{3.3}$$

Profits of seller i are then given by

$$\Pi(p_i,p^*) = p_i D^D(p_i;p^*). \tag{3.4}$$

Proposition 3.1. *Let f and h be continuously differentiable densities on the supports $[a_F,b_F]$ and $[a_H,b_H]$. Furthermore, let $0 < s < E(t_i) - a$. When the symmetric Nash* *equilibrium exists its price under directed search for any* $\lambda \in (0,1)$ *is given by:*

$$p^* = \frac{-1}{n\frac{\partial D^D}{\partial p_i}(p^*, p^*)}, \tag{3.5}$$

with equilibrium profits $\Pi^* = \frac{p^*}{n}$. *A sufficient condition for existence of the equilibrium is* $f'(\varepsilon) \geq 0 \ \forall \varepsilon$. *In the case of* $\lambda = 1$ *or* $s \geq E(t_i) - a$, *sellers charge infinite prices in equilibrium.*

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It is necessary that $s > 0$, otherwise consumers search freely, and a situation of perfect information occurs in which prices will drop to zero. The condition $s < E(t_i) - a$ is imposed to ensure that there is search on the market. If this requirement would not be met, search cost would be so high that it is unbeneficial for consumers to search onward if all sellers charge the same price, even if the match value at the current seller is the worst possible. Anderson and Renault (1999) already pointed to the similarity of this setting to that of Diamond (1971): for any price set by the competitors, a seller can increase its price without affecting its demand. Hence, in equilibrium all sellers set infinite prices. I find that this situation is analogous to that of the case of $\lambda = 1$under the directed search regime.

Equation (3.5) is found by equating the derivative of (3.4) at p^* to zero. The remainder of the proof that (3.5) gives a global maximum when $f'(\varepsilon) \geq 0 \ \forall \varepsilon$ is presented in Appendix 3.A. The Diamond-type argument that prices and profits go to infinity when $s \geq E(t_i) - a$ is trivial and is omitted.

The condition $f'(\varepsilon) \geq 0 \ \forall \varepsilon$ is satisfied by the uniform distribution, and more generally for any distribution $F(\varepsilon) = \varepsilon^{\kappa}$ with $\kappa \geq 1$. Establishing the existence of equilibrium for the somewhat more general conditions given by Anderson and Renault (1999) is not my central concern. This chapter's aim is to quantify the value of information.

Note that the Proposition does not treat the case $\lambda = 0$. In that case consumers do not value the CPMV and basing their search order upon it makes no sense. The model then collapses to that of random search, which is treated in the next section and acts as a benchmark.

3.4 Benchmark

I now derive equilibrium prices under the assumption of random search. The price in the symmetric Nash equilibrium under this scenario is denoted by p^r. The demand of seller i under the random search rule can be derived similar fashion as above and in the expression below, where $\Delta_r = p_i - p^r$, it is presented. More details can be found in Anderson and Renault (1999).

$$\begin{aligned} D^r(p_i, p^r) &= \frac{1}{n}\int\limits_{-\infty}^{\infty}\left[1 - F\left(\frac{\bar{x} + \Delta_r - \lambda v_i}{1-\lambda}\right)\right] h(v_i)\mathrm{d}v_i \\ &+ \frac{1}{n}\sum_{l=2}^{n}\left[\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}\left[1 - F\left(\frac{\bar{x} + \Delta_r - \lambda v_i}{1-\lambda}\right)\right] F\left(\frac{\bar{x} - \lambda v_j}{1-\lambda}\right)\right. \\ &\left.\cdot\, R(\bar{x}, b_H)^{l-2} h(v_i)\mathrm{d}v_i h(v_j)\mathrm{d}v_j\right] + \int\limits_{-\infty}^{\bar{x}+\Delta_r} M(t_i - \Delta_r)^{n-1} m(t_i)\mathrm{d}t_i \\ &= \frac{1 - M(\bar{x} + \Delta_r)}{n}\frac{1 - M(\bar{x})^n}{1 - M(\bar{x})} + \int\limits_{-\infty}^{\bar{x}+\Delta_r} M(t_i - \Delta_r)^{n-1} m(t_i)\mathrm{d}t_i, \end{aligned} \tag{3.6}$$

as $R(\bar{x}, b_H) = M(\bar{x})$. Here $\bar{x}$ solves $g(x, b_H) = s$. Since $s > 0$, $\bar{x} < b$.

Profits for seller i under the random search regime are:

$$\Pi^r(p_i, p^r) = p_i D^r(p_i; p^r). \tag{3.7}$$

Proposition 3.2. *Let f and h be continuously differentiable densities on the supports $[a_F, b_F]$ and $[a_H, b_H]$. Furthermore, let $0 < s < E(t_i) - a$. When the symmetric Nash equilibrium exists its price under random search for any $\lambda \in [0, 1]$ is given by:*

$$p^r = \frac{-1}{n\frac{\partial D^r}{\partial p_i}(p^r; p^r)}, \tag{3.8}$$

with equilibrium profits $\Pi^{*r} = \frac{p^r}{n}$. A sufficient condition for existence is of the equilibrium is $f'(\varepsilon_i) \geq 0$. In the case of $s \geq E(t_i) - a$, sellers charge infinite prices in equilibrium.

A proof of this Proposition can be found in Anderson and Renault (1999).

3.5 Comparative statics

Due to the complexity of the model I restrict attention to standard uniformly distributed CPMV and IPMV and 2 sellers in the remainder of this section. Moreover, I assume $\lambda \in (0, 0.5]$ and that search costs are sufficiently small: $s \leq \frac{1-\lambda}{8}$. This last condition ensures that there will be no consumers who refrain from searching beyond the first sampled seller even before they know the realization of the IPMV. Appendix 3.B includes a proof of this last claim and presents several Lemma's in which $\hat{x}$, $\bar{x}$, $\frac{\partial D^D}{\partial p_i}(p^*, p^*)$ and $\frac{\partial D^r}{\partial p_i}(p^r, p^r)$ are derived explicitly for uniformly distributed match values and 2 sellers. Using these Lemma's and numerical methods the following set of Propositions is derived.

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Proposition 3.3. $p^* \geq p^r$.

The Proposition states that sellers charge higher prices when consumers use directed instead of random search. Under directed search consumers will first visit the seller that offers the product with the highest expected utility, which reduces demand elasticity. This results is opposite to the effect of prominence is Armstrong et al. (2009). There the prominent seller sets a lower price than in the case of random search because it faces more elastic demand as all consumers sample it first. Moreover, in my model sellers are symmetric, leading to equal prices, while in Armstrong et al. (2009) non-prominent sellers set a price higher than under random search, and therfore higher than the prominent seller's price.

One interpretation of the Proposition is that disclosing product's horizontal attributes leads to higher prices. This is also found Meurer and Stahl (1994) and Anderson and Renault (2009), although in different settings. These papers consider buyers who observe prices and are not able to search for the best product match. However, some consumers might be informed about product characteristics through advertising, which leads to higher prices.

Proposition 3.4. *The equilibrium price p^* is non-decreasing in search costs.*

The intuition behind this result is that when search costs are higher it is more costly for consumers to inspect the option at the next seller. Sellers know that consumers are thus less likely to continue searching and will charge a higher price. For the equilibrium price under random search, p^r, a similar result can be established, see Anderson and Renault (1999).

Proposition 3.5.

(a) p^* is decreasing in the CPMV weight λ if $s < \tilde{s}$, where $\tilde{s}$ is a function of λ.

(b) p^* is increasing in λ if $s > \tilde{s}$.

This Proposition is the result of two mechanisms which work in opposite directions. To see this, first consider the impact of λ upon the prices under random search, p^r. A change in λ does not effect the search decision of consumers directly, as they search random. However, the change does affect the distribution of the total match value, $t_i = (1-\lambda)\varepsilon_i + \lambda v_i$. An increase in $\lambda \in (0, 0.5)$ leads to a lower variance of t_i, but leaves its expected value unaffected. A lower variation in the total match value means less product differentation. As Anderson and Renault (1999) shows, less differentiation leads to less monopoly power for sellers and thus lower prices. Under directed search an increase of λ has the additional effect that consumers are better informed: for a larger part of the match value they know that the first seller they visit offers the best option. Hence, the expected return from searching decreases in λ in that case and leads to upward pressure on prices. This effect only dominates the other effect when search costs are sufficiently large. After all, when search costs are small consumers are likely to search onward, even if the weight on the CPMV is increased.

The discussion above shows that λ can not directly be interpreted as the amount of product information made available to consumers, as a change of it alters the distribution of the match value as well. Hence, λ should be fixed and the CPMV might be used to determine the search order or not.

Corollary 3.1. $\Pi^* \geq \Pi^{*r}$.

Proof of Corollary 3.1. All sellers have an equal market share (1/2) in each equilibrium, as they charge the same price as their competitor, whether search is random or when it depends on the OPMV. The result now follows, since $p^* \geq p^r$ by Proposition 3.3. ∎

The Propositions in this section are established for uniformly distributed match values and a duopoly. Model complexity prevents me to generalize these results or establish them analytically. However, I do allow for more general distributions and more than 2 sellers when analyzing the welfare effects of product information in the next section.

3.6 Welfare

Throughout this section I assume that $\lambda \neq 0$ and $s > 0$. It is convenient to define

$$t_i^m = (1-\lambda)\varepsilon_i + \lambda v_m, \text{ with } v_m \geq v_j \;\; \forall j.$$

Proposition 3.6. *Consumers search strictly less under directed search than under random search as in each stage of the search process the consumers is less likely to continue search.*

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Proof of Proposition 3.6. First consider the decision whether to continue search when a consumer has already arrived at at least two sellers. Recall $t_i = (1-\lambda)\varepsilon_i + \lambda v_i$. Notice that the maximum of $n-1$ independent random variables with distribution H has $(n-1)H(v_m)^{n-2}h(v_m)$ as its probability density function. Under directed search a consumer continues search when $t_i \leq \hat{x}$, which happens with probability

$$\int_{-\infty}^{\infty}\int_{-\infty}^{v_m} F\left(\frac{\hat{x}-\lambda v_i}{1-\lambda}\right) h(v_i)\mathrm{d}v_i (n-1)H(v_m)^{n-2}h(v_m)\mathrm{d}v_m. \tag{3.9}$$

In Appendix 3.A it is shown that $\hat{x} \leq \bar{x}$ for all v_m, therefore it follows that this probability equals at most

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F\left(\frac{\bar{x}-\lambda v_i}{1-\lambda}\right) h(v_i)\mathrm{d}v_i (n-1)H(v_m)^{n-2}h(v_m)\mathrm{d}v_m = \int_{-\infty}^{\infty} F\left(\frac{\bar{x}-\lambda v_i}{1-\lambda}\right) h(v_i)\mathrm{d}v_i, \tag{3.10}$$

which is exactly the probability of continuing search under random search. Hence, under directed search the consumer who has already left the first seller continues searching at most as often as in the random search regime.

Now consider the probability of searching under in the different settings when a consumer has arrived at the first seller. In the random search case one has a match value of t_i while under the directed regime one has a match value t_i^m. Straightforwardly, one obtains $\Pr[t_i \leq \bar{x}] \geq \Pr[t_i^m \leq \bar{x}] \geq \Pr[t_i^m \leq \hat{x}]$. The first inequality holds strictly if $v_i \neq v_m$. So, a consumer under the directed regime is less likely to continue search from the first firm than under the random search rule. Therefore, expected search expenditures are strictly lower under the directed regime. ■

Proposition 3.7. *The expected total match value a consumer obtains under directed search is higher than under random search.*

Proof of Proposition 3.7. The possibility of consumers returning to a seller after visiting all sellers is irrelevant for the analysis. When such an event happens, the expected match value a consumer obtains is the same in case of random and directed search because she can choose from the same set of sellers. Let v_m, the highest CPMV on the market, be fixed. Define

$$Z = \lambda v_m + E[(1-\lambda)\varepsilon_j] \quad \text{and} \quad W = E[\lambda v_i + (1-\lambda)\varepsilon_i | v_i < v_m].$$

Z is the expected match value of the seller with the highest CPMV, say seller j. W is the expected match value at the other sellers. Notice that $Z \geq W$. The likelihood of a consumer arriving at seller j equals 1 in case of directed search, as this seller will be sampled first. Under random search it is strictly lower as a consumer might sample another seller first and buy there. To show that the expected match value under directed search is higher it thus suffices to show that consumers are less likely to continue search from seller j. Under random search this happens with probability $\Pr[t_j^m \leq \bar{x}]$ as consumers do not realize this is the seller with the highest CPMV. Under consumer-dependent prominece it is lower and equals $\Pr[t_j^m \leq \hat{x}]$. ■

For an individual consumer it is optimal to minimize the search costs and maximize the expected match value. In both cases directed search outperforms random search, implying that this will be the strategy followed by consumers.

It is informative to compare the above results to those in Armstrong et al. (2009). They find that consumers also search less compared to the case of random search, although for a different reason. In their model a consumer is more likely to buy at the first seller because it charges a lower price than its rivals while in my model she is more likely to do so because the match value is on average higher. In their framework a consumer obtains on average a lower match value than when she would search randomly because the lower price at the first seller induces some consumers not to continue searching although it would be socially efficient. In my model the preferences of a consumer affect which seller is visited first, which actually leads to the opposite effect of higher expected match value in equilibrium.

As prices are merely transfers in a covered market, total welfare is higher under directed search than under random search.[3]

Corollary 3.2. *Total welfare under directed search is higher than under random search.*

This finding is opposite to that in Armstrong et al. (2009) where total welfare is lower than in the case where consumers visit sellers in a random order. The reason is that in my model consumers are better matched to sellers than under random search while in their model the matching is equally bad and consumers search less than what is socially efficient. This observataion leads to a conclusion that is opposite to that in Armstrong et al (2009) as well. Suppose there is a profit maximizing platform that provides a link between consumers and sellers. If the platform is able to extract total welfare by charging both consumers and sellers, it will have an incentive to introduce direct consumers based upon their preferences.

Having considered total welfare and profits I now turn to consumer welfare. I derive an expression for the expected match value and search expenditures under directed and random search. I start with directed search. Notice that $nH(v_m)^{n-1}$ is the density of the maximum of n CPMV's and $(n-1)H(v_m)^{n-2}$ that of the maximum of $n-1$ CPMV's. Hence, the expected match from buying at the first seller equals

$$E(t_i^m|t_i^m > \hat{x}) = \int_{-\infty}^{\infty} \int_{\frac{\hat{x}-\lambda v_m}{1-\lambda}}^{\infty} \frac{[(1-\lambda)\varepsilon + \lambda v_m]f(\varepsilon)}{1-F\left(\frac{\hat{x}-\lambda v_m}{1-\lambda}\right)} nH(v_m)^{n-1} \mathrm{d}\varepsilon h(v_m) \mathrm{d}v_m,$$

while the expected match from buying directly from the l^{th} visited seller, $l \in \{2, \dots, n\}$, equals

$$E(t_i|t_i > \hat{x}) = \int_{-\infty}^{\infty} \int_{-\infty}^{v_m} \int_{\frac{\hat{x}-\lambda v}{1-\lambda}}^{\infty} \frac{[(1-\lambda)\varepsilon + \lambda v]f(\varepsilon)}{1-F(\frac{\hat{x}-\lambda v}{1-\lambda})} h(v)(n-1)H(v_m)^{n-2} h(v_m) \mathrm{d}\varepsilon \mathrm{d}v \mathrm{d}v_m.$$

When a consumer returns to an earlier visited seller her expected match is the expectation of the maximum of n match values. By considering first the case that $\max_{i=1,\dots n}\{t_i\} \neq t^m$ and then the case that $\max_{i=1,\dots n}\{t_i\} = t^m$ we find that this

[3] If the model would allow for nonpurchase than the higher equilibrium prices under directed search might possibly offset the positive effects of lower search cost and higher expected match values and lead to a welfare loss. However, when search costs are sufficiently small or when there is not much product differentiation, prices will approach those of perfect competition and the market will be covered.

expected match value equals

$$E\left(\max_{i=1,\dots n}\{t_i\}\middle|t_i<\hat{x}\right)=\int_{-\infty}^{\infty}\int_{-\infty}^{v_m}\int_{-\infty}^{\frac{\hat{x}-\lambda v}{1-\lambda}}\frac{[(1-\lambda)\varepsilon_i+\lambda v_i]f(\varepsilon_i)}{F\left(\frac{\hat{x}-\lambda v}{1-\lambda}\right)}$$

$$\cdot\left[\int_{-\infty}^{v_m}F\left(\frac{(1-\lambda)\varepsilon_i+\lambda v_i-\lambda v_j}{1-\lambda}\right)h(v_j)\mathrm{d}v_j\right]^{n-2}F\left(\frac{(1-\lambda)\varepsilon_i+\lambda v_i-\lambda v_m}{1-\lambda}\right)\mathrm{d}\varepsilon_i h(v_i)\mathrm{d}v_i h(v_m)\mathrm{d}v_m$$

$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\frac{\hat{x}-\lambda v_m}{1-\lambda}}\frac{[(1-\lambda)\varepsilon_i+\lambda v_m]f(\varepsilon_i)}{F\left(\frac{\hat{x}-\lambda v_m}{1-\lambda}\right)}\left[\int_{-\infty}^{v_m}F\left(\frac{(1-\lambda)\varepsilon_i+\lambda v_m-\lambda v_j}{1-\lambda}\right)h(v_j)\mathrm{d}v_j\right]^{n-1}\mathrm{d}\varepsilon_i h(v_m)\mathrm{d}v_m.$$

Here $f(\varepsilon)/F\left(\frac{\hat{x}-\lambda v}{1-\lambda}\right)$ is the distribution of ε_i conditional on $t_i<\hat{x}$ and terms as $F\left(\frac{(1-\lambda)\varepsilon_i+\lambda v_i-\lambda v_j}{1-\lambda}\right)$ ensure $t_i\geq t_j$.

There are $nQ_1(p^*,p^*)$ consumers who buy from the first seller they arrive at. Here $Q_1(p^*,p^*)$ is given in equation (3.1), this term is multiplied with n as there are n sellers active who can have the highest CPMV for a consumer. Similarly, the number of consumers buying from seller $l\in\{2,\dots,n\}$ without having visited it before is given by $(n-l+1)Q_l(p^*,p^*)$, where $Q_l(p^*,p^*)$ is given in (3.2). Finally, the number of comebacks on the market is given by

$$\Pr\left(t_i<\hat{x}\ \forall i\in\{1,\dots n\}\right)=\int_{-\infty}^{\infty}F\left(\frac{\hat{x}-\lambda v_m}{1-\lambda}\right)R(\hat{x},v_m)^{n-1}h(v_m)\mathrm{d}v_m.$$

Using these expressions I find that total welfare under random search is given by

$$SW_D=nQ_1(p^*,p^*)[-s+E(t_i^m|t_i^m>\hat{x})]+\sum_{l=2}^{n}(n-l+1)Q_l(p^*,p^*)[-ls+E(t_i|t_i>\hat{x})]$$
$$+\Pr\left(t_i<\hat{x}\ \forall i\in\{1,\dots n\}\right)\left[-ns+E\left(\max_{i=1,\dots n}\{t_i\}|t_i<\hat{x}\right)\right].$$

Social welfare resulting from random search can be derived in a similar fashion and equals

$$SW_R=[1-M(\bar{x})]\sum_{l=1}^{n}M(\bar{x})^{l-1}[-ls+E(t_i|t_i\geq\bar{x})]+M(\bar{x})^n\left[-ns+E\left(\max_{i=1,\dots n}\{t_i\}\middle|t_i<\bar{x}\right)\right].$$

Corollary 3.2 established that $SW_D>SW_R$. However, Proposition 3.3 shows that $p^*\geq p^r$. So, on the one hand, consumers pay higher prices under directed search, but on the other hand, they obtain a higher match values and spend less on search costs. I evaluate $SW_D-p^*-(SW_R-p^r)$ to determine whether consumers are worse off under directed search. I apply numerical methods due to the complexity of the expression. Again I will restrict attention to sufficiently small search costs, uniformly distributed CPMV's and IPMV's, $\lambda \in (0, 0.5]$ and $n = 2$. Appendix 3.B presents for this setting $\frac{\partial D^D}{\partial p_i}(p^*, p^*)$ and $\frac{\partial D^r}{\partial p_i}(p^r, p^r)$ from which p^* and p^r directly follow. I find the following result.

Proposition 3.8. *Consumer surplus under directed search is lower than under random search.*

3.7 Conclusion

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In this chapter I incorporated the idea that an intermediating platform between consumers and sellers may base the order in which the sellers are listed on the preferences of a consumer. Based on this order consumers first sample the seller where they expect to find the product that gives them the highest utility. The model fits into the literature on consumer search with differentiated products and contributes to the literature on position auctions as well.

Under directed search I find that prices and profits are higher as compared to the case of random search. The driving force behind this result is that sellers realize that consumers will first sample the seller at which they expect to find the product that fits their tastes the best. This gives them some monopoly power over these consumers, which is exploited in equilibrium. Disclosing product information through the platform effectively allows sellers to differentiate their products and charge higher prices. In line with existing literature I find that prices are increasing in search costs, as higher search costs reduce demand elasticity. In the extreme case when all product characteristics are displayed on the platform the Diamond paradox arises again.

Total welfare is higher under directed search than in case of random search. This is the result of two effects. First, consumers obtain on average a higher match value because they start their search at a seller which offers a product that, in certain dimensions, fits their tastes the best. Second, consumers are less likely to search onward in each stage of the process when their preferences influences which seller to sample first. This is because consumers know they will never find a product that fits their preferences better for certain aspects than the one offered by the first seller. One implication of this result is that when a profit maximizing platform is able to extract total welfare by charging both consumers and sellers, it will have an incentive to suggest sellers based upon consumer preferences.

Consumer welfare turns out to be lower under directed search, as the positive effects of better matches and lower search expenditures are outweighed by the higher prices.

Future research might extend the presented model and allow the entire listing order to depend upon consumer preferences. Such an extension is far from trivial since it takes away the stationary character of the search process as the expected benefit of continuing search will then depend upon the available product information of the next best seller.

3.A General distributions

First I give a useful Lemma and its proof.

Lemma 1. $g(x, v^m) = \int_{-\infty}^{v^m} \int_{\frac{x-\lambda v_j}{1-\lambda}}^{\infty} ((1-\lambda)\varepsilon_j + \lambda v_j - x) f(\varepsilon_j) d\varepsilon_j \frac{h(v_j)}{H(v^m)} dv_j$ *is decreasing in* x *and increasing in* v^m*. In addition,* $\hat{x} \leq \bar{x}$ *where* $\hat{x}$ *solves* $g(x, v^m) = s$ *and* $\bar{x}$ *solves* $g(x, b) = s$.

Proof of Lemma 1. By inspecting the expression of $g(x, v^m)$ it immediately follows that it is decreasing in x. Since

$$\begin{aligned}
\frac{\partial g(x, v^m)}{\partial v^m} &= \int_{\frac{x-\lambda v^m}{1-\lambda}}^{\infty} ((1-\lambda)\varepsilon_j + \lambda v^m - x) f(\varepsilon_j) \mathrm{d}\varepsilon_j \frac{h(v^m)}{H(v^m)} \\
&- \int_{-\infty}^{v^m} \int_{\frac{x-\lambda v_j}{1-\lambda}}^{\infty} ((1-\lambda)\varepsilon_j + \lambda v_j - x) f(\varepsilon_j) \mathrm{d}\varepsilon_j \frac{h(v_j) h(v^m)}{H(v^m)^2} \mathrm{d}v_j \\
&> \int_{\frac{x-\lambda v^m}{1-\lambda}}^{\infty} ((1-\lambda)\varepsilon_j + \lambda v^m - x) f(\varepsilon_j) \mathrm{d}\varepsilon_j \frac{h(v^m)}{H(v^m)} \\
&- \int_{-\infty}^{v^m} \int_{\frac{x-\lambda v^m}{1-\lambda}}^{\infty} ((1-\lambda)\varepsilon_j + \lambda v^m - x) f(\varepsilon_j) \mathrm{d}\varepsilon_j \frac{h(v_j) h(v^m)}{H(v^m)^2} \mathrm{d}v_j = 0
\end{aligned}$$

$g(x, v^m)$ is increasing in v^m. Combining these results gives $\hat{x} \leq \bar{x}$. ■

Proof of Proposition 3.1. I show that profit function is concave for $\lambda \in (0,1)$, the case $\lambda = 1$ will be treated later. Define

$$u_i = \varepsilon_i - \frac{\Delta}{1-\lambda} \text{ and } \Theta = \left\{ u_i : u_i \in \left[a_F - \frac{\Delta}{1-\lambda}, \frac{\hat{x} - \lambda v_m}{1-\lambda} \right] \right\}. \tag{3.11}$$

Using this and using integration by parts one finds[5]:

$$\begin{aligned}
&\int_{-\infty}^{\infty} \int_{-\infty}^{\frac{\hat{x}+\Delta-\lambda v_m}{1-\lambda}} \left(\int_{-\infty}^{v_m} F\left(\frac{(1-\lambda)\varepsilon_i + \lambda v_m - \Delta - \lambda v_j}{1-\lambda} \right) h(v_j)\mathrm{d}v_j \right)^{n-1} f(\varepsilon_i)\mathrm{d}\varepsilon_i h(v_m)\mathrm{d}v_m \\
= &\int_{-\infty}^{\infty} \int_{\Theta} \left(1 - F\left(u_i + \frac{\Delta}{1-\lambda}\right)\right) \mathrm{d}\left(\int_{-\infty}^{v_m} F\left(u_i + \frac{\lambda v_m - \lambda v_j}{1-\lambda}\right) h(v_j)\mathrm{d}v_j \right)^{n-1} h(v_m)\mathrm{d}v_m \\
&+ \int_{-\infty}^{\infty} \left(\int_{-\infty}^{v_m} F\left(a_F + \frac{\lambda v_m - \Delta - \lambda v_j}{1-\lambda}\right) h(v_j)\mathrm{d}v_j \right)^{n-1} h(v_m)\mathrm{d}v_m \\
&- \int_{-\infty}^{\infty} \left[1 - F\left(\frac{\hat{x} - \lambda v_m + \Delta}{1-\lambda}\right)\right] \left(\int_{-\infty}^{v_m} F\left(\frac{\hat{x} - \lambda v_j}{1-\lambda}\right) h(v_j)\mathrm{d}v_j \right)^{n-1} h(v_m)\mathrm{d}v_m.
\end{aligned} \tag{3.12}$$

Define

$$T(u, v_i, v_m) = \left[\int_{-\infty}^{v_m} F\left(u_i + \frac{\lambda v_i - \lambda v_j}{1-\lambda}\right) h(v_j)\mathrm{d}v_j \right]^{n-2} F\left(u_i + \frac{\lambda v_i - \lambda v_m}{1-\lambda}\right).$$

Applying this definition and using integration by parts one finds[6]:

$$\begin{aligned}
&\sum_{l=2}^{n} \int_{-\infty}^{\infty} \int_{-\infty}^{v_m} \int_{-\infty}^{\frac{\hat{x}+\Delta-\lambda v_i}{1-\lambda}} \left[\int_{-\infty}^{v_m} F\left(\frac{(1-\lambda)\varepsilon_i + \lambda v_i - \Delta - \lambda v_j}{1-\lambda} \right) h(v_j)\mathrm{d}v_j \right]^{n-2} \\
&\qquad \cdot F\left(\frac{(1-\lambda)\varepsilon_i + \lambda v_i - \Delta - \lambda v_m}{1-\lambda} \right) f(\varepsilon_i)\mathrm{d}\varepsilon_i h(v_i)\mathrm{d}v_i h(v_m)\mathrm{d}v_m \\
= &\sum_{l=2}^{n} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[\int_{\Theta} \left(1 - F\left(u_i + \frac{\Delta}{1-\lambda}\right)\right) \mathrm{d}T(u_i, v_i, v_m) - \left(1 - F\left(\frac{\hat{x} - \lambda v_i + \Delta}{1-\lambda}\right)\right) \right.
\end{aligned}$$

[5]Start the computation with the first term after the equality sign. Apply integration by parts on the inner integral of this expression. That is, calculate $\int_\Theta \alpha(u_i)\beta'(u_i)dx = \alpha(\frac{\hat{x}-\lambda v_m}{1-\lambda})\beta(\frac{\hat{x}-\lambda v_m}{1-\lambda}) - \alpha(a_F - \frac{\Delta}{1-\lambda})\beta(a_F - \frac{\Delta}{1-\lambda}) - \int_\Theta \beta(u_i)\alpha'(u_i)\mathrm{d}u_i$ with $\beta'(u_i) = \mathrm{d}\left(\int_{-\infty}^{v_m} F\left(u_i + \frac{\lambda v_m - \lambda v_j}{1-\lambda}\right) h(v_j)\mathrm{d}v_j \right)^{n-1}$ and $\alpha(u_i) = \left(1 - F\left(u_i + \frac{\Delta}{1-\lambda}\right)\right)$. Bringing the term $\int_\Theta \beta(u_i)\alpha'(u_i)\mathrm{d}u_i$ to the other side of the equality sign then gives the result. In Anderson and Renault (1999) a similar approach is used to calculate $D_A(p_i, p^*)$ and $D_B(p_i, p^*)$ at page 733.

[6]Use the same approach as in footnote 5.

$$\cdot T\left(\frac{\hat{x}-\lambda v_i}{1-\lambda}, v_i, v_m\right) + T\left(a_F - \frac{\Delta}{1-\lambda}, v_i, v_m\right)\bigg] h(v_i)\mathrm{d}v_i h(v_j)\mathrm{d}v_m. \quad (3.13)$$

Using (3.12) and (3.13) to rewrite demand from comebacks in (3.3) and applying some algebra gives

$$D^D(p_i, p^*) = \int_{-\infty}^{\infty} \left[D_1(p_i, v_m) + D_2(p_i, v_m) + D_3(p_i, v_m) + D_4(p_i, v_m)\right] h(v_m)\mathrm{d}v_m, \quad (3.14)$$

where

$$\begin{aligned}
D_1(p_i, v_m) &= \left[1 - F\left(\frac{\hat{x}+\Delta-\lambda v_m}{1-\lambda}\right)\right]\left[H(v_m)^{n-1} - \left(\int_{-\infty}^{v_m} F\left(\frac{\hat{x}-\lambda v_j}{1-\lambda}\right) h(v_j)\mathrm{d}v_j\right)^{n-1}\right] \\
D_2(p_i, v_m) &= \int_{-\infty}^{v_m}\left[1 - F\left(\frac{\hat{x}+\Delta-\lambda v_i}{1-\lambda}\right)\right]\sum_{l=2}^{n}\left[H(v_m)^{n-l}\left(\int_{-\infty}^{v_m} F\left(\frac{\hat{x}-\lambda v_j}{1-\lambda}\right) h(v_j)\mathrm{d}v_j\right)^{l-2}\right. \\
&\quad \left. - \left(\int_{-\infty}^{v_m} F\left(\frac{\hat{x}-\lambda v_j}{1-\lambda}\right) h(v_j)\mathrm{d}v_j\right)^{n-2}\right] F\left(\frac{\hat{x}-\lambda v_m}{1-\lambda}\right) h(v_i)\mathrm{d}v_i \\
D_3(p_i, v_m) &= \int_{\Theta}\left(1 - F\left(u_i + \frac{\Delta}{1-\lambda}\right)\right)\mathrm{d}\left(\int_{-\infty}^{v_m} F\left(u_i + \frac{\lambda v_m - \lambda v_j}{1-\lambda}\right) h(v_j)\mathrm{d}v_j\right)^{n-1} \\
&\quad + \left(\int_{-\infty}^{v^m} F\left(a_F + \frac{\lambda v^m - \Delta - \lambda v_j}{1-\lambda}\right) h(v_j)\mathrm{d}v_j\right)^{n-1} \\
D_4(p_i, v_m) &= \sum_{l=2}^{n}\int_{-\infty}^{v_m}\left[\int_{a_F-\frac{\Delta}{1-\lambda}}^{\frac{\hat{x}-\lambda v_i}{1-\lambda}}\left(1 - F\left(u_i + \frac{\Delta}{1-\lambda}\right)\right)\mathrm{d}T(u_i, v_i, v_m)\right. \\
&\quad \left. + T\left(a_F - \frac{\Delta}{1-\lambda}, v_i, v_m\right)\right] h(v_i)\mathrm{d}v_i.
\end{aligned}$$

Notice that:

$$H(v_m)^{n-1} - \left(\int_{-\infty}^{v_m} F\left(\frac{\hat{x}-\lambda v_j}{1-\lambda}\right) h(v_j)\mathrm{d}v_j\right)^{n-1} \geq H(v_m)^{n-1} - \left(\int_{-\infty}^{v_m} h(v_j)\mathrm{d}v_j\right)^{n-1} \geq 0, \quad (3.15)$$

and similarly for all $l \in \{2, 3, \ldots, n\}$:

$$H(v_m)^{n-l}\left(\int_{-\infty}^{v_m} F\left(\frac{\hat{x}-\lambda v_j}{1-\lambda}\right) h(v_j)\mathrm{d}v_j\right)^{l-2} - \left(\int_{-\infty}^{v_m} F\left(\frac{\hat{x}-\lambda v_j}{1-\lambda}\right) h(v_j)\mathrm{d}v_j\right)^{n-2} \geq 0. \quad (3.16)$$

It follows that every term in (3.14), and therefore demand, is non-increasing in p_i.

3

Let $\pi_k(p_i,p^*) = p_i D_k(p_i, v_m)$, for $k \in \{1,2,3,4\}$. Then seller i her profits are $\Pi(p_i,p^*) = \int\limits_{-\infty}^{\infty} \sum_{k=1}^{4} \pi_k(p_i,p^*)h(v_m)\mathrm{d}v_m$. I show $\pi_k(p_i,p^*)$ is concave for all $k \in \{1,2,3,4\}$.
By (3.15) π_1 is concave whenever

$$-\frac{2}{1-\lambda} f\left(\frac{\hat{x}+\Delta-\lambda v_m}{1-\lambda}\right) - \frac{p_i}{(1-\lambda)^2} f'\left(\frac{\hat{x}+\Delta-\lambda v_m}{1-\lambda}\right) \leq 0.$$

As $f'(\varepsilon) \geq 0 \ \forall \varepsilon$ this inequality holds. π_2 can be rewritten as a positive constant times $\int\limits_{-\infty}^{v_m} \left[1 - F\left(\frac{\hat{x}+\Delta-\lambda v_i}{1-\lambda}\right)\right] h(v_i)\mathrm{d}v_i$. Therefore $\frac{\partial^2 \pi_2}{\partial p_i^2}$ is proportional to:

$$\int\limits_{-\infty}^{v_m} \left[-\frac{2}{1-\lambda} f\left(\frac{\hat{x}+\Delta-\lambda v_i}{1-\lambda}\right) - \frac{p_i}{(1-\lambda)^2} f'\left(\frac{\hat{x}+\Delta-\lambda v_i}{1-\lambda}\right)\right] h(v_i)\mathrm{d}v_i.$$

Since $f'(\varepsilon) \geq 0 \ \forall \varepsilon$ this expression is non-positive. Therefore π_2 is concave. $\frac{\partial^2 \pi_3}{\partial p_i^2}$ equals

$$\int\limits_{\Theta} \left[-\frac{2}{1-\lambda} f\left(u_i + \frac{\Delta}{1-\lambda}\right) - \frac{p_i}{(1-\lambda)^2} f'\left(u_i + \frac{\Delta}{1-\lambda}\right)\right]$$
$$\cdot \mathrm{d}\left(\int\limits_{-\infty}^{v_m} F\left(u_i + \frac{\lambda v_m - \lambda v_j}{1-\lambda}\right) h(v_j)\mathrm{d}v_j\right)^{n-1} - p_i \frac{(n-1)f(a_F)}{(1-\lambda)^2}$$
$$\cdot \left(\int\limits_{-\infty}^{v_m} F\left(a_F + \frac{\lambda v_m - \Delta - \lambda v_j}{1-\lambda}\right) h(v_j)\mathrm{d}v_j\right)^{n-2} \left(\int\limits_{-\infty}^{v_m} f\left(a_F + \frac{\lambda v_m - \Delta - \lambda v_j}{1-\lambda}\right) h(v_j)\mathrm{d}v_j\right).$$

By again using $f'(\varepsilon) \geq 0 \ \forall \varepsilon$ it straightforwardly follows that $\frac{\partial^2 \pi_3}{\partial p_i^2} \leq 0$. $\frac{\partial^2 \pi_4}{\partial p_i^2}$ is equal to

$$\sum_{l=2}^{n} \int\limits_{-\infty}^{v_m} \int\limits_{\Theta} \left[-\frac{2}{1-\lambda} f\left(u_i + \frac{\Delta}{1-\lambda}\right) - \frac{p_i}{(1-\lambda)^2} f'\left(u_i + \frac{\Delta}{1-\lambda}\right)\right] \mathrm{d}T(u_i, v_i, v_m) h(v_i)\mathrm{d}v_i.$$
$$-\sum_{l=2}^{n} \int\limits_{-\infty}^{v_m} \frac{p_i}{(1-\lambda)^2} f(a_F)(n-2) \left[\int\limits_{-\infty}^{v_m} F\left(a_F + \frac{\lambda v_i - \Delta - \lambda v_j}{1-\lambda}\right) h(v_j)\mathrm{d}v_j\right]^{n-3}$$
$$\cdot \left[\int\limits_{-\infty}^{v_m} f\left(a_F + \frac{\lambda v_i - \Delta - \lambda v_j}{1-\lambda}\right) h(v_j)\mathrm{d}v_j\right] F\left(a_F + \frac{\lambda v_i - \Delta - \lambda v_m}{1-\lambda}\right) h(v_i)\mathrm{d}v_i$$
$$-\sum_{l=2}^{n} \int\limits_{-\infty}^{v_m} \frac{p_i f(a_F)}{(1-\lambda)^2} \left[\int\limits_{-\infty}^{v_m} F\left(a_F + \frac{\lambda v_i - \Delta - \lambda v_j}{1-\lambda}\right) h(v_j)\mathrm{d}v_j\right]^{n-2}$$
$$\cdot f\left(a_F + \frac{\lambda v_i - \Delta - \lambda v_m}{1-\lambda}\right) h(v_i)\mathrm{d}v_i.$$

All terms in the expression are non-positive since $f'(\varepsilon) \geq 0 \ \forall \varepsilon$, which leads to the conclusion that $\pi_4(p_i, p^*)$ is concave.

Because the sum of concave functions is concave it follows by these arguments that a sufficient condition for the symmetric Nash equilibrium to exists is $f'(\varepsilon) \geq 0 \ \forall \varepsilon$ when $\lambda \in (0, 1)$.

Now suppose $\lambda = 1$. Then consumers perfectly observe the entire match value in the first stage of the game, and the IPMV is zero. However, they still have to search for the best price. An argument that is analogous to that of Diamond shows that the best response to any market price is an increase of the price. Prices will explode under the covered market assumption. ■

3

3.B Uniformly distributed match values

To derive an explicit expressions for $\frac{\partial D^D}{\partial p_i}(p^*, p^*)$ and $\frac{\partial D^r}{\partial p_i}(p^r, p^r)$ it is convenient to write out the density of a convex combination of two uniformly distributed variables.

The convolution of two uniform distributions. Let X and Y be uniformly distributed on $[0, b_F]$ and $[0, b_H]$, respectively. The density of $Z = \lambda Y + (1-\lambda)X$, denoted by m, is given by:

$$m(z) = \begin{cases} 0 & \text{if } z \leq 0 \\ \frac{z}{\lambda b_H (1-\lambda) b_F} & \text{if } z \in (0, \min\{\lambda b_H, (1-\lambda) b_F\}] \\ \frac{\min\{\lambda b_H, (1-\lambda) b_F\}}{\lambda b_H (1-\lambda) b_F} & \text{if } z \in [\min\{\lambda b_H, (1-\lambda) b_F, \max\{\lambda b_H, (1-\lambda) b_F\}] \\ \frac{\lambda b_H + (1-\lambda) b_F - z}{\lambda b_H (1-\lambda) b_F} & \text{if } z \in [\max\{\lambda b_H, (1-\lambda) b_F\}, \lambda b_H + (1-\lambda) b_F) \\ 0 & \text{if } z \geq \lambda b_H + (1-\lambda) b_F. \end{cases}$$

Moreover, the following Lemma will be useful.

Lemma 2. *For uniformly distributed match values and $\lambda \in (0, 0.5]$ the following statements hold.*

(a) When $s \in [\frac{\lambda^2 v_m^2}{6(1-\lambda)}, \frac{1-\lambda}{8}]$ then $\hat{x} = 1 - \lambda + \frac{1}{2}\lambda v_m - \sqrt{2(1-\lambda)s - \frac{\lambda^2 v_m^2}{12}}$.

(b) When $s \in [\frac{\lambda^2}{6(1-\lambda)}, \frac{1-\lambda}{8}]$ then $\bar{x} = 1 - \frac{1}{2}\lambda - \sqrt{2(1-\lambda)s - \frac{\lambda^2}{12}}$.

(c) When $s < \frac{\lambda^2 v_m^2}{6(1-\lambda)}$ then $\hat{x} = 1 - \lambda + \lambda v_m - (6 v_m \lambda (1-\lambda) s)^{1/3}$.

(d) When $s < \frac{\lambda^2}{6(1-\lambda)}$ then $\bar{x} = 1 - (6\lambda(1-\lambda)s)^{1/3}$.

When $s < \frac{1-\lambda}{8}$ then $\hat{x} \geq \lambda v_m$ for all v_m and the decision to continue search will depend upon the realization of a consumer's IPMV.

Proof of Lemma 2. Suppose $\hat{x} \in [\lambda v_m, 1-\lambda]$, then $g(x, v_m)$ reduces to

$$g(x,v_m) = \int_0^{v_m} \int_{\frac{x-\lambda v_j}{1-\lambda}}^{1} ((1-\lambda)\varepsilon_j + \lambda v_j - x)\mathrm{d}\varepsilon_j \frac{1}{v_m}\mathrm{d}v_j = \int_0^{v_m} \frac{(1-\lambda+\lambda v_j - x)^2}{2(1-\lambda)v_m}\mathrm{d}v_j$$

$$= \frac{1}{6(1-\lambda)}\left(\lambda^2 v_m^2 - 3\lambda v_m x - 3\lambda^2 v_m + 3\lambda v_m + 3x^2 + 6\lambda x - 6x + 3\lambda^2 - 6\lambda + 3\right).$$

Equating this to s and solving for x gives $\hat{x} = 1-\lambda+\frac{1}{2}\lambda v_m - \sqrt{2(1-\lambda)s - \frac{\lambda^2 v_m^2}{12}}$. As $\hat{x} \leq 1-\lambda$ this puts the condition $s \geq \frac{\lambda^2 v_m^2}{6(1-\lambda)}$ on s. However, when s is sufficiently large it might happen that $1-\lambda+\frac{1}{2}\lambda v_m - \sqrt{2(1-\lambda)s - \frac{\lambda^2 v_m^2}{12}} < \lambda v_m$ for sufficiently small v_m, contradicting $\hat{x} \geq \lambda v_m$. In that case there are some consumers who will decide not to continue search based upon their CPMV, independent of their IPMV. I assume $s \leq \frac{1-\lambda}{8}$ to keep the model tractable. The expression for $\bar{x}$ and the conditions imposed on s for case (b) are found by taking $v_m = 1$ in part (a).

Now suppose $\hat{x} > 1-\lambda$, then one finds

$$g(x,v_m) = \int_{\frac{x-1+\lambda}{\lambda}}^{v_m} \frac{(1-\lambda+\lambda v_j - x)^2}{2(1-\lambda)v_m}\mathrm{d}v_j = \frac{(1-\lambda+\lambda v_m - x)^3}{6v_m\lambda(1-\lambda)}.$$

Solving $g(\hat{x}, v_m) = s$ gives $\hat{x} = 1-\lambda+\lambda v_m - (6v_m\lambda(1-\lambda)s)^{1/3}$. Setting $v_m = 1$ in this expression gives $\bar{x}$. The conditions on s for this case now straightforwardly follow. ■

The following Lemma gives explicit versions of $\frac{\partial D^D}{\partial p_i}(p^*, p^*)$ and $\frac{\partial D^r}{\partial p_i}(p^r, p^r)$.

Lemma 3. *Let $\check{v} = \frac{1}{\lambda}\sqrt{6(1-\lambda)s}$. For uniformly distributed match values, $s \leq \frac{1-\lambda}{8}$ and $\lambda \in (0, 0.5]$ the following statements hold.*

(a) When $s \leq \min\{\frac{1-\lambda}{8}, \frac{\lambda^2}{6(1-\lambda)}\}$ then

$$\frac{\partial D^D}{\partial p_i}(p^*,p^*) = -\frac{1}{2(1-\lambda)} - \int_{\check{v}}^{1} \frac{-2\lambda v_m(6v_m\lambda(1-\lambda)s)^{1/3} + (1-\lambda)^2}{2\lambda(1-\lambda)^2}\mathrm{d}v_m$$
$$- \int_0^{\check{v}} \frac{4v_m - 2 - 3\lambda v_m + 2\lambda - 3\lambda v_m^2 - 2(2v_m-1)\sqrt{2(1-\lambda)s - \frac{\lambda^2 v_m^2}{12}}}{2(1-\lambda)^2}\mathrm{d}v_m. \quad (3.17)$$

(b) When $s \in \left(\frac{\lambda^2}{6(1-\lambda)}, \frac{1-\lambda}{8}\right)$*, a situation which can not occur when* λ *sufficiently large, then*

$$\frac{\partial D^D}{\partial p_i}(p^*, p^*) = -\frac{1}{2(1-\lambda)}$$

$$-\int_0^1 \frac{4v_m - 2 - 3\lambda v_m + 2\lambda - 3\lambda v_m^2 - 2(2v_m - 1)\sqrt{2(1-\lambda)s - \frac{\lambda^2 v_m^2}{12}}}{2(1-\lambda)^2} \mathrm{d}v_m. \tag{3.18}$$

(c) When $s \leq \min\{\frac{1-\lambda}{8}, \frac{\lambda^2}{6(1-\lambda)}\}$*, then* $\bar{x} = 1 - (6\lambda(1-\lambda)s)^{1/3}$ *and*

$$\frac{\partial D^r}{\partial p_i}(p^r, p^r) = -\frac{\bar{x}^3 - 3\bar{x}^2 + 3\bar{x} - 16\lambda^3 + 12\lambda^2 - 1}{12\lambda^2(1-\lambda)^2}. \tag{3.19}$$

(d) When $s \in \left(\frac{\lambda^2}{6(1-\lambda)}, \frac{1-\lambda}{8}\right)$*, then* $\bar{x} = 1 - \frac{1}{2}\lambda - \sqrt{2(1-\lambda)s - \frac{\lambda^2}{12}}$ *and*

$$\frac{\partial D^r}{\partial p_i}(p^r, p^r) = -\frac{6(1+\bar{x}) - 11\lambda}{12(1-\lambda)^2}. \tag{3.20}$$

Proof of Lemma 3. By Lemma 2 $\hat{x} = 1 - \lambda + \lambda v_m - (6v_m\lambda(1-\lambda)s)^{1/3}$ for $v_m \geq \check{v}$ and $\hat{x} = 1 - \lambda + \frac{1}{2}\lambda v_m - \sqrt{2(1-\lambda)s - \frac{\lambda^2 v_m^2}{12}}$ for $v_m < \check{v}$. First assume $\check{v} \leq 1$, that is, $s \leq \frac{\lambda^2}{6(1-\lambda)}$.

The derivative of the first three lines of (3.3) with respect to p_i, at $\Delta = 0$, is

$$-\int_{-\infty}^{\infty} H(v_m)^{n-1} f\left(\frac{\hat{x} - \lambda v_m}{1-\lambda}\right) \frac{1}{1-\lambda} h(v_m)\mathrm{d}v_m - \sum_{l=2}^{n} \left[\int_{-\infty}^{\infty}\int_{-\infty}^{v_m} f\left(\frac{\hat{x} - \lambda v_i}{1-\lambda}\right)\right.$$

$$\left.\cdot\frac{1}{1-\lambda} H(v_m)^{n-l} F\left(\frac{\hat{x} - \lambda v_m}{1-\lambda}\right) R(\hat{x}, v_m)^{l-2} h(v_i)\mathrm{d}v_i h(v_m)\mathrm{d}v_m\right]. \tag{3.21}$$

Note $v_m > \check{v}$ if and only if $\hat{x} > 1 - \lambda$. Therefore (3.21) becomes for $n = 2$

$$-\frac{1}{2(1-\lambda)} - \left[\int_{\check{v}}^{1}\int_{\frac{\hat{x}-1+\lambda}{\lambda}}^{v_m} \mathrm{d}v_i \frac{1}{1-\lambda}\frac{\hat{x} - \lambda v_m}{1-\lambda}\mathrm{d}v_m + \int_0^{\check{v}}\int_0^{v_m} \mathrm{d}v_i \frac{1}{1-\lambda}\frac{\hat{x} - \lambda v_m}{1-\lambda}\mathrm{d}v_m\right]. \tag{3.22}$$

For $n = 2$ and uniformly distributed match values the derivative of the last four lines of (3.3) with respect to p_i at $\Delta = 0$ becomes:

$$\frac{1}{1-\lambda}\int_{-\infty}^{\infty}\left(\int_{-\infty}^{v_m} F\left(\frac{\hat{x} - \lambda v_j}{1-\lambda}\right) h(v_j)\mathrm{d}v_j\right) f\left(\frac{\hat{x} - \lambda v^m}{1-\lambda}\right) h(v_m)\mathrm{d}v_m$$

$$
-\frac{1}{1-\lambda}\int_{-\infty}^{\infty}\int_{-\infty}^{\frac{\hat{x}-\lambda v_m}{1-\lambda}}\left(\int_{-\infty}^{v_m} f\left(\frac{(1-\lambda)\varepsilon_i+\lambda v_m-\lambda v_j}{1-\lambda}\right)h(v_j)\mathrm{d}v_j\right)f(\varepsilon_i)\mathrm{d}\varepsilon_i h(v_m)\mathrm{d}v_m
$$

$$
+\frac{1}{1-\lambda}\int_{-\infty}^{\infty}\int_{-\infty}^{v_m} F\left(\frac{\hat{x}-\lambda v_m}{1-\lambda}\right)f\left(\frac{\hat{x}-\lambda v_i}{1-\lambda}\right)h(v_i)\mathrm{d}v_i h(v_m)\mathrm{d}v_m
$$

$$
-\frac{1}{1-\lambda}\int_{-\infty}^{\infty}\int_{-\infty}^{v_m}\int_{-\infty}^{\frac{\hat{x}-\lambda v_i}{1-\lambda}} f\left(\frac{(1-\lambda)\varepsilon_i+\lambda v_i-\lambda v_m}{1-\lambda}\right)f(\varepsilon_i)\mathrm{d}\varepsilon_i h(v_i)\mathrm{d}v_i h(v_m)\mathrm{d}v_m.
$$

This equals

$$
\frac{1}{1-\lambda}\int_0^{\check{v}}\int_0^{v_m}\frac{\hat{x}-\lambda v_j}{1-\lambda}\mathrm{d}v_j\mathrm{d}v_m+\frac{1}{1-\lambda}\int_{\check{v}}^{1}\left(\frac{\hat{x}-1+\lambda}{\lambda}+\int_{\frac{\hat{x}-1+\lambda}{\lambda}}^{v_m}\frac{\hat{x}-\lambda v_j}{1-\lambda}\mathrm{d}v_j\right)\mathrm{d}v_m
$$

$$
-\frac{1}{1-\lambda}\int_0^{\check{v}}\int_0^{\frac{\hat{x}-\lambda v_m}{1-\lambda}}\int_0^{v_m}\mathrm{d}v_j\mathrm{d}\varepsilon_i\mathrm{d}v_m-\frac{1}{1-\lambda}\int_{\check{v}}^{1}\int_0^{\frac{\hat{x}-\lambda v_m}{1-\lambda}}\int_{\frac{(1-\lambda)\varepsilon-(1-\lambda)+\lambda v_m}{\lambda}}^{v_m}\mathrm{d}v_j\mathrm{d}\varepsilon_i\mathrm{d}v_m
$$

$$
+\frac{1}{1-\lambda}\int_{\check{v}}^{1}\frac{\hat{x}-\lambda v_m}{1-\lambda}\left(v_m-\frac{\hat{x}-1+\lambda}{\lambda}\right)dv_m+\frac{1}{1-\lambda}\int_0^{\check{v}}\frac{\hat{x}-\lambda v_m}{1-\lambda}\mathrm{d}v_m
$$

$$
-\frac{1}{1-\lambda}\int_0^{1}\int_0^{v_m}\int_{\frac{\lambda v_m-\lambda v_i}{1-\lambda}}^{\frac{\hat{x}-\lambda v_i}{1-\lambda}}\mathrm{d}\varepsilon_i\mathrm{d}v_i\mathrm{d}v_m. \tag{3.23}
$$

Adding (3.23) to (3.22) and some algebra leads to the expression given in part (a) of the Lemma. Part (b) can be derived in a similar fashion and by noticing that $\check{v}>1$ when $s>\frac{\lambda^2}{6(1-\lambda)}$.

Now consider random search. The derivative of (3.6) with respect to p_i at $\Delta=0$ is for $n=2$:

$$
-\frac{1}{2(1-\lambda)}\left[1+\int_{-\infty}^{\infty}F\left(\frac{\bar{x}-\lambda v_j}{1-\lambda}\right)h(v_j)\mathrm{d}v_j\right]\int_{-\infty}^{\infty}f\left(\frac{\bar{x}-\lambda v_i}{1-\lambda}\right)h(v_i)\mathrm{d}v_i
$$

$$
+M(\bar{x})m(\bar{x})-\int_{-\infty}^{\bar{x}}m(t_i)^2\mathrm{d}t_i. \tag{3.24}
$$

First suppose $s>\frac{\lambda^2}{6(1-\lambda)}$, then $\bar{x}\in[\lambda,1-\lambda)$. Hence, $f\left(\frac{\bar{x}-\lambda v_i}{1-\lambda}\right)=1$ and $F\left(\frac{\bar{x}-\lambda v_j}{1-\lambda}\right)=\frac{\bar{x}-\lambda v_j}{1-\lambda}$ for all $v_i,v_j\in[0,1]$. When one uses $m(t_i)$ for the uniform distributions given at the start of this Appendix one finds after some calculations:

$$M(\bar{x})m(\bar{x}) - \int_{-\infty}^{\bar{x}} m(t_i)^2 \mathrm{d}t_i = \frac{2\bar{x} - \lambda}{2(1-\lambda)^2} - \frac{3\bar{x} - 2\lambda}{3(1-\lambda)^2} = \frac{\lambda}{6(1-\lambda)^2}.$$

Combining these findings with (3.24) and Lemma 2 results in part (d) of the Lemma. Next consider $s \leq \frac{\lambda^2}{6(1-\lambda)}$, then $\bar{x} \geq 1 - \lambda$. Hence, $\int_{-\infty}^{\infty} f\left(\frac{\bar{x}-\lambda v_i}{1-\lambda}\right) h(v_i)\mathrm{d}v_i = \frac{1-\bar{x}}{\lambda}$ and $M(\bar{x}) = \int_{-\infty}^{\infty} F\left(\frac{\bar{x}-\lambda v_j}{1-\lambda}\right) h(v_j)\mathrm{d}v_j = \frac{2\bar{x}-1+2\lambda-2\lambda^2-\bar{x}^2}{2\lambda(1-\lambda)}$. Furthermore, using $m(t_i)$ as given for the uniform case at the start of this Appendix gives $\int_{-\infty}^{\bar{x}} m(t_i)^2\mathrm{d}t_i = \frac{3\lambda^2-4\lambda^3+3\bar{x}-3\bar{x}^2+\bar{x}^3-1}{3\lambda^2(1-\lambda)^2}$. Combining these findings with (3.24) and Lemma 2 results in part (c) of the Lemma. ■

Part II

Switching

Chapter **4**

Winning back the unfaithful while exploiting the loyal: Retention offers and heterogeneous switching costs*

4.1 Introduction

In subscription-type markets, e.g. those for credit cards, cable, telecom, and insurance, firms are often willing to offer a better deal to consumers who indicate that they want to cancel their subscription. These offers are known as retention offers, as firms make them in an attempt to retain fickle consumers. Consumers' reactions to these practices differ. Some seem largely unaware of it, or at least unwilling to exploit such offers. Others actively chase them, sharing details of current offers on websites like flyertalk.com.

In this chapter, we analyze retention offers. We assume that there are two types of consumers; those with relatively low, and those with relatively high switching costs. Firms can use retention offers to screen consumers with low switching costs. Consumers that have already gone through the trouble of obtaining an offer from a competing firm, signal that they have low switching costs and hence are likely to switch. Retention offers then effectively serve as a mechanism to price discriminate against consumers with high switching costs.

We thus focus on cases where consumers cancel their current subscription in favor of a competitor. For example, in the UK, Ofcom (2010) reports that in e.g. mobile telephony, consumers that want to switch have to contact their current provider and

*This chapter is based on Haan and Siekman (2015b).

request a code which they must communicate to their new provider to complete the switch. However, when applying for such a code, the current provider can, and often does, make a retention offer. Indeed, this chapter was inspired by a similar experience. One of the authors who switched to a cheaper car insurance still received a renewal from the old insurer. He phoned them, the company apologized, asked why he cancelled his policy, and what price the new insurer charged. It then offered a price slightly below that – which he was willing to accept. It is exactly this experience that we try to model in this chapter.

We study a two-period model with two firms located at the endpoints of a Hotelling line. In the second period, firms set prices based on buying behavior in period 1. In particular, firm B can try to poach consumers from firm A by charging them a lower price. Once a consumer indicates that she intends to switch from A to B to take advantage of that poaching price, however, firm A can make a retention offer. In the equilibrium of our model, low-switching-cost consumers strategically solicit offers from the competing firm to secure a retention offer from their current provider – even if they have no intention to switch. Soliciting offers requires costly effort, and high-switching-cost consumers do not find making that effort worthwhile. Hence, using retention offers allows firms to price discriminate between the two types.

We find that the possibility of retention offers increases prices. Prices for loyal consumers increase, as this pool of consumers is less likely to switch on average. But poaching prices increase as well; as low-cost consumers have already incurred part of their switching costs, they become easier to poach. Equilibrium prices in the first period also increase. As competition for consumers with low switching costs is fiercer in the second period, firms are less eager to attract these consumers in period 1. The welfare effects are ambiguous. Firms are better off, while consumers are worse off. The latter applies to all individual high-switching-cost consumers, and to consumers as a whole. The effect on individual low-cost consumers is ambiguous.

This chapter clearly fits in the literature on behavior-based price discrimination. Classic references in this field include Chen (1997), Fudenberg and Tirole (2000) and Taylor (2003), that all look at multi-period models in which firms can base the price they charge on a consumer's purchase history. Chen and Pearcy (2010) allow consumer tastes to evolve over the course of the game. Gehrig, Shy and Stenbacka (2011) study the welfare effects of behavior-based price discrimination in the context of entry deterrence. Yet, none of these papers allows for retention offers. This chapter adds to the literature on switching costs, of which overviews can be found in Klemperer (1995) and Farrell and Klemperer (2007).

Two recent papers, developed independently from this chapter, do look at retention offers. Gnutzmann (2013) extends Chen (1997) by looking at retention offers in a model with homogeneous products and $N \geq 2$ firms. In the second period, consumers can readily observe loyalty prices, poaching prices and retention prices, but have to exert effort 1 to secure the poaching price and effort $\alpha < 1$ to secure the retention price. Consumers differ in their cost of effort, i.e. their switching costs. In this paper, different from this chapter, first-period prices do not affect second-period actions, as consumers only learn their switching costs after the first period. Esteves (2014) looks at a model that is similar to the one presented here. She extends the Fudenberg and Tirole (2000) with the possibility of retention offers. Crucially, however, she does not allow consumers to strategically solicit an offer from the competing firm, in an attempt to obtain a better deal from their current supplier. In her model, consumers do not rationally foresee that retention offers will be made.

Finally note that retention offers differ from price-matching policies, in which a supplier is always willing to match a lower price of a competitor. Such price-matching policies do not depend on purchase behavior of consumers. Also, in the model presented here, we will see that the equilibrium retention price is actually higher than the poaching price offered by the competitor, simply because the consumer has already revealed a preference for this supplier by her past buying behavior. Price-matching policies are studied in e.g. Arbatskaya, Hviid and Shaffer (2004) and Corts (1997).

This chapter is organized as follows. First, section 4.2 introduces the model. Section 4.3 considers a benchmark in which there are no retention offers, but there is poaching and heterogeneous switching costs. The model with retention offers is studied in section 4.4. We study the effects of the possibility of retention offers in Section 4.5 and conclude in Section 4.6.

4.2 The model

A unit mass of consumers is uniformly distributed on a Hotelling line. Transportation costs are normalized to 1. Firms A and B are located at 0 and 1 respectively and face marginal costs c. There are 2 periods. Consumers have unit demand in each period, and willingness-to-pay r, gross of transportation costs. The market is fully covered. Firms and consumers use a common discount factor $\delta \in (0, 1)$.

There are two types of consumers: those with high switching costs z_H, and those with low switching costs $z_L < z_H$. The share of low types is given by $\lambda \in (0, 1)$, independent of location. Switching costs are incurred if a consumer switches suppliers in period 2. Switching costs consist of two elements. First, a consumer has to **prepare** for a switch, for example by securing an offer from the competing supplier. Second, she has to **effectuate** the switch, for example by actually signing a contract with the new supplier.

For the analysis, it is crucial that actions taken to prepare for a switch satisfy three conditions. First, they have to involve **sunk costs** to the consumer. Second, they have to be **revocable**, in the sense that after incurring preparation costs, the consumer still has the option to stick to her original supplier. Third, the costly actions have to be **observable** to her original supplier. For example, consider a consumer that considers to switch car insurers. She secures an offer from the other supplier, and then makes a phone call to her current supplier to cancel her contract. The costs involved with these actions are sunk. However, the switch is revocable: she may still change her mind and stay with the current supplier. Finally, the current supplier observes that this consumer has contacted her and, possibly, also that she has secured a competing offer. Hence, all three conditions are satisfied.

We denote the costs for a type $i \in \{L, H\}$ to prepare for a switch as z_i^1, and the additional costs to perform the switch as z_i^2. We assume that both types of switching costs are higher for the high types. Thus $z_H^1 > z_L^1$ and $z_H^2 > z_L^2$, while $z_L = z_L^1 + z_L^2$ and $z_H = z_H^1 + z_H^2$.

The timing of the game is as follows. In **period 1**, A and B simultaneously set prices p_A^1 and p_B^1, respectively. Consumers observe these prices, and decide where to buy in the first period. A fraction $\hat{x}_i^1$ of type i consumers, to be determined endogenously, will buy from firm A. The other $1-\hat{x}_i^1$ will buy from firm B. We will refer to the consumers that buy from A in period 1 as segment A, and to the consumers that buy from B in period 1 as segment B. In **period 2**, the following sequence of events unfolds. In the first stage, firms A and B simultaneously each set 2 prices, observable to everyone. Firm A charges a **loyalty price** p_{AA}^2 to consumers that bought from A in period 1, and a **poaching price** p_{AB}^2 to consumers that bought from B. Similarly, B sets prices p_{BB}^2 and p_{BA}^2. In the second stage, each consumer decides whether she incurs preparation costs z_i^1. If she does, her original supplier can observe this and can make a retention offer. The retention offer of firm A is denoted p_A^R, that of firm B is p_B^R.

For the analysis that follows to be valid, we need to impose some parameter restrictions. These restrictions imply that in the benchmark model without retention offers for any value of λ some, but not all, low type consumers, and some, but not all, high type consumers are poached in the second period. Moreover, we want the same thing to be true in the model with retention offers. As we will show in the analysis below, this requires the following parameter restrictions to hold;

$$z_L < 1; \tag{4.1}$$

$$z_H < 1/3 + 2z_L/3; \tag{4.2}$$

$$z_H < 1/2 + z_L^2/2. \tag{4.3}$$

4

4.3 Benchmark: no retention offers

Preliminaries We first consider a benchmark without retention offers. In that case, the separation of total switching costs into preparation costs and effectuation costs is immaterial. The timing of this simplified game is thus as follows. In **period 1**, A and B simultaneously set p_A^1 and p_B^1, and a fraction $\hat{x}_i^1$ of type i consumers buys from A. These consumers comprise segment A, the others segment B. In **period 2**, A and B simultaneously set poaching prices and loyalty prices. We look for a symmetric equilibrium and solve with backward induction.

Second period In equilibrium at least some type i consumers in segment A will be tempted by the poaching price of firm B. The second period will then have some $\hat{x}_{Ai}^2 < \hat{x}_i^1$ consumers again choosing for firm A, while the remaining $\hat{x}_i^1 - \hat{x}_{Ai}^2$ switch to

B. Something similar holds for consumers in segment B. The indifferent types i on segments A and B are given by

$$\hat{x}^2_{Ai} = \tfrac{1}{2}\left(1 + p^2_{BA} - p^2_{AA} + z_i\right); \qquad \hat{x}^2_{Bi} = \tfrac{1}{2}\left(1 + p^2_{BB} - p^2_{AB} - z_i\right), \tag{4.4}$$

provided that these expressions are strictly between 0 and the relevant $\hat{x}^1_i$. Parameter restrictions (4.1) and (4.2) assure that that is the case in equilibrium.[1]

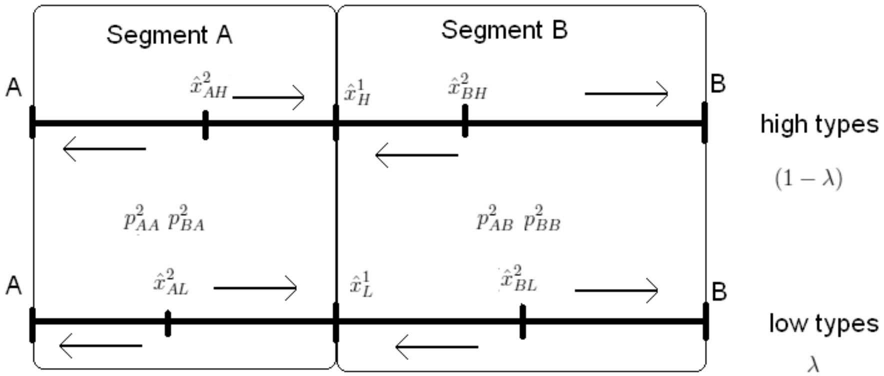

Figure 4.1: Market segmentation in both periods, Benchmark.

Figure 4.1 depicts this situation. The top panel reflects consumers with high switching costs, the bottom panel those with low switching costs. In period 1, those to the left of $\hat{x}^1_i$ buy from firm A and thus comprise segment A.[2] Those to the right of $\hat{x}^1_i$ buy from B and comprise segment B. In period 2, those in segment A that are located to the left of $\hat{x}^2_{AH}$ will buy from firm A (as reflected by the arrow), while those to the right will buy from B. Something similar applies to those in segment B. As $z_H > z_L$, we have from (4.4) that $x^2_{AH} > x^2_{AL}$ and $x^2_{BH} > x^2_{BL}$: as their switching costs are higher, fewer high types will switch in period 2.

[1] To have $\hat{x}^2_{Ai} > 0$, we need $(2 + 3z_i - 2\bar{z}) > 0$, hence $2\bar{z} < 2 + 3z_i$. We want this to be satisfied for all λ. It is most restrictive for $\lambda = 0$, in which case it yields

$$2z_H < 2 + 3z_i. \tag{4.5}$$

For the high types, this is always satisfied. For the low types, it requires $2z_H - 3z_L < 2$.

In a symmetric equilibrium, we will have $\hat{x}^1_i = 1/2$. For the second period, we thus need $\hat{x}^2_{Ai} < 1/2$, hence $2 + 3z_i - 2\bar{z} < 3$, so $3z_i - 2\bar{z} < 1$. We want this to be satisfied for all λ. It is most restrictive for $\lambda = 1$, in which case it yields $3z_i - 2z_L < 1$. For the low types, this requires (4.1). For the high types, it requires $3z_H - 2z_L < 1$, or (4.2). Note that if this is satisfied, (4.5) is satisfied as well.

[2] We will show below that $\hat{x}^1_H = \hat{x}^1_L$, as is depicted in the figure.

Second-period profits for firm A are given by

$$\begin{aligned}
\Pi_A^2 &= \Pi_{AA}^2 + \Pi_{AB}^2 \\
&\equiv (p_{AA}^2 - c)\left[\lambda \hat{x}_{AL}^2 + (1-\lambda)\hat{x}_{AH}^2\right] \\
&\quad +(p_{AB}^2 - c)\left[\lambda(\hat{x}_{BL}^2 - \hat{x}_L^1) + (1-\lambda)(\hat{x}_{BH}^2 - \hat{x}_H^1)\right], \qquad (4.6)
\end{aligned}$$

where Π_{AA}^2 (the second line) reflects total profits from loyal consumers, and Π_{AB}^2 (the third line) total profits from consumers that are poached. Similarly, firm B's profits are given by

$$\begin{aligned}
\Pi_B^2 &= \Pi_{BB}^2 + \Pi_{BA}^2 \\
&\equiv (p_{BA}^2 - c)\left[\lambda(\hat{x}_L^1 - \hat{x}_{AL}^2) + (1-\lambda)(\hat{x}_H^1 - \hat{x}_{AH}^2)\right] \\
&\quad +(p_{BB}^2 - c)\left[\lambda(1 - \hat{x}_{BL}^2) + (1-\lambda)(1 - \hat{x}_{BH}^2)\right]. \qquad (4.7)
\end{aligned}$$

For ease of exposition, we define $\bar{z}$ as the weighted average of switching costs in the population, and $\hat{x}^1$ as the weighted average location of indifferent consumers in period 1:

$$\bar{z} \equiv \lambda z_L + (1-\lambda) z_H, \qquad (4.8)$$

$$\hat{x}^1 \equiv \lambda \hat{x}_L^1 + (1-\lambda)\hat{x}_H^1. \qquad (4.9)$$

Plugging in the expressions from (4.4) into the second line of (4.6), we have

$$\Pi_{AA}^2 = \frac{1}{2}(p_{AA}^2 - c)\left[1 + p_{BA}^2 - p_{AA}^2 + \bar{z}\right]. \qquad (4.10)$$

Similarly, for firm B, from the second line of (4.7), and (4.4),

$$\Pi_{BA}^2 = (p_{BA}^2 - c)\left[\hat{x}^1 - \frac{1}{2}\left(1 + p_{BA}^2 - p_{AA}^2 + \bar{z}\right)\right], \qquad (4.11)$$

Maximizing (4.10) with respect to p_{AA}^2 and (4.11) with respect to p_{BA}^2 yields the following reaction functions:

$$p_{AA}^2 = \tfrac{1}{2}\left(1 + p_{BA}^2 + c + \bar{z}\right); \qquad p_{BA}^2 = \tfrac{1}{2}\left(2\hat{x}^1 - 1 + p_{AA}^2 + c - \bar{z}\right).$$

Solving the system gives:

$$p_{AA}^2 = c + \tfrac{1}{3}(1 + 2\hat{x}^1 + \bar{z}); \qquad p_{BA}^2 = c + \tfrac{1}{3}(4\hat{x}^1 - 1 - \bar{z}). \qquad (4.12)$$

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We then immediately have

$$\hat{x}^2_{Ai} = \frac{1}{6}\left(1 + 2\hat{x}^1 + 3z_i - 2\bar{z}\right) \tag{4.13}$$

and

$$\Pi^2_{AA} = \tfrac{1}{18}(1 + 2\hat{x}^1 + \bar{z})^2; \qquad \Pi^2_{BA} = \tfrac{1}{18}(4\hat{x}^1 - 1 - \bar{z})^2. \tag{4.14}$$

On segment B, we can do a similar analysis. Here

$$\begin{aligned}
\Pi^2_{BB} &= \frac{1}{2}(p^2_{BB} - c)\left[1 + p^2_{AB} - p^2_{BB} + \bar{z}\right], \\
\Pi^2_{AB} &= (p^2_{AB} - c)\left[1 - \hat{x}^1 - \frac{1}{2}\left(1 + p^2_{AB} - p^2_{BB} + \bar{z}\right)\right].
\end{aligned}$$

Hence

$$p^2_{AB} = c + \tfrac{1}{3}(3 - 4\hat{x}^1 - \bar{z}); \qquad \Pi^2_{AB} = \tfrac{1}{18}(3 - 4\hat{x}^1 - \bar{z})^2. \tag{4.15}$$

First period We now solve for the first period. Consumers are forward-looking and rationally take into account the events that will unfold in the second period. A consumer that is indifferent between A and B in period 1 thus anticipates that, whatever she chooses, she will switch in period 2. Denoting the discount factor by δ, the indifferent type i located at $\hat{x}^1_i$ has

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$$\begin{aligned}
r - \hat{x}^1_i - p^1_A \quad + \quad & \delta(r - (1 - \hat{x}^1_i) - p^2_{BA} - z_i) \\
= \quad & r - (1 - \hat{x}^1_i) - p^1_B + \delta(r - \hat{x}^1_i - p^2_{AB} - z_i),
\end{aligned} \tag{4.16}$$

where the left-hand side gives her total lifetime utility if she chooses A in period 1, while the right-hand side gives that of choosing B in period 1. Note that switching costs z_i drop out of this equality; either way, in equilibrium this consumer will always incur switching costs in period 2, so these do not affect $\hat{x}^1_i$. This immediately implies $\hat{x}^1_L = \hat{x}^1_H = \hat{x}^1$. Solving (4.16) then gives

$$\hat{x}^1 = \frac{1 + p^1_B - p^1_A - \delta(1 + p^2_{BA} - p^2_{AB})}{2(1-\delta)}. \tag{4.17}$$

Substituting second-period equilibrium prices from (4.12) and (4.15) and solving for $\hat{x}^1$ yields

$$\hat{x}^1 = \frac{1}{2} + \frac{3(p^1_B - p^1_A)}{6 + 2\delta}. \tag{4.18}$$

In the first period, firm A sets p_A^1 as to maximize total discounted profits

$$\begin{aligned}\Pi_A &= (p_A^1 - c)\hat{x}^1 + \delta \Pi_{AA}^2 + \delta \Pi_{AB}^2 \\ &= (p_A^1 - c)\hat{x}^1 + \frac{\delta}{18}(1 + 2\hat{x}^1 + \bar{z})^2 + \frac{\delta}{18}(3 - 4\hat{x}^1 - \bar{z})^2. \end{aligned} \tag{4.19}$$

Taking the derivative with respect to p_A^1 :

$$\frac{\partial \Pi_A}{\partial p_A^1} = (p_A^1 - c)\frac{\partial \hat{x}^1}{\partial p_A^1} + \hat{x}^1 + \frac{2\delta}{9}\left(1 + 2\hat{x}^1 + \bar{z}\right)\frac{\partial \hat{x}^1}{\partial p_A^1} - \frac{4\delta}{9}\left(3 - 4\hat{x}^1 - \bar{z}\right)\frac{\partial \hat{x}^1}{\partial p_A^1}.$$

A symmetric equilibrium requires $p_A^1 = p_B^1$ hence $\hat{x} = \frac{1}{2}$. From (4.18), we have $\frac{\partial \hat{x}^1}{\partial p_A^1} = \frac{-3}{6+2\delta}$. Hence, the first-order condition becomes

$$\frac{1}{2} - \frac{3}{6+2\delta}\left(p_A^1 - c + \frac{2\delta \bar{z}}{3}\right) = 0.$$

This yields equilibrium prices

$$p_A^1 = p_B^1 = c + 1 + \frac{\delta}{3}\left(1 - 2\bar{z}\right).$$

We thus have the following:[3]

[3] The second-period profit functions (4.6) and (4.7) are clearly concave – provided that firms set prices such that the indifferent high and low type consumers are both strictly between 0 and 1/2 in equilibrium. Yet, it may still be profitable to do a large defection. We will show that that is not the case. As in the main text, we focus on segment A.

First consider firm B. It can defect to a price p_{BA}^2 that is so high that it only sells to the low types. In equilibrium, that requires setting p_{BA}^2 such that $\hat{x}_{AH} \geq 1/2$, or $1 + p_{BA}^2 - p_{AA}^2 + z_H \geq 1$. This implies setting $p_{BA}^2 = p_{AA}^2 - z_H$. Its profits are then given by

$$\pi_{BL}^2 = \lambda\left(p_{BA}^2 - c\right)\left[\frac{1}{2} - \frac{1}{2}\left(1 + p_{BA}^2 - p_{AA}^2 + z_L\right)\right],$$

which is maximized by setting $p_{BA}^2 = \frac{1}{2}\left(p_{AA}^2 + c - z_L\right)$. At $p_{BA}^2 = p_{AA}^2 - z_H$, these profits are decreasing whenever $p_{AA}^2 - z_H > \frac{1}{2}\left(p_{AA}^2 + c - z_L\right)$, hence if $z_H\left(\lambda + 5\right) - z_L\left(\lambda + 3\right) < 2$. This is most restrictive for $\lambda = 1$, so we need $z_H < \frac{1}{3} + \frac{2}{3}z_L$ which is exactly (4.2). Hence, we are on the downward sloping part of π_L. Therefore such a defection cannot be profitable.

Alternatively, firm B could set p_{BA}^2 so low that we serve all the low types, so $\hat{x}_{AL}^2 = 0$. That implies setting

$$p_{BA}^2 = p_{AA}^2 - z_L - 1 = c + \frac{1}{3}\left(2 + \bar{z}\right) - z_L - 1 = c + \frac{1}{3}\left(\bar{z} - 1\right) - z_L < c,$$

which is clearly unprofitable.

Now consider firm A. First, it can defect by setting p_{AA}^2 so high that it only sells to the high types. That requires setting p_{AA}^2 such that $\hat{x}_{AL} \leq 0$ or $p_{AA}^2 \geq 1 + p_{BA}^2 + z_L$. Its profits are then given by

$$\pi_{AH}^2 = \frac{1}{2}\left(1 - \lambda\right)\left(p_{AA}^2 - c\right)\left[1 + p_{BA}^2 - p_{AA}^2 + z_H\right],$$

which is maximized by setting $p_{AA}^2 = \frac{1}{2}\left(1 + p_{BA}^2 + c + z_H\right)$. At $p_{AA}^2 = 1 + p_{BA}^2 + z_L$, these profits are decreasing whenever $1 + p_{BA}^2 + c + z_H < 2p_{BA}^2 + 2z_L + 2$ or $-2z_L - \frac{4}{3} + z_H + \bar{z}/3 < 0$, which is always the case. Hence, such a defection cannot be profitable.

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Proposition 4.1. *In the benchmark without retention offers, equilibrium first-period, loyalty and poaching prices are given by*

$$\begin{aligned} p_1^{bm} &= c+1+\frac{\delta}{3}\left(1-2\bar{z}\right); \\ p_{loyal}^{bm} &= c+\frac{1}{3}(2+\bar{z}); \\ p_{poach}^{bm} &= c+\frac{1}{3}(1-\bar{z}). \end{aligned} \tag{4.20}$$

Equilibrium profits are given by

$$\Pi^{bm}=\frac{1}{2}+\frac{1}{18}\left(8\delta-2\bar{z}\delta\left(2-\bar{z}\right)\right).$$

In a model with standard Hotelling competition, without poaching or switching costs, we would have $p=p^h\equiv c+1$ in each period. From (4.1), we have $\bar{z}<1$, hence $p^h>p_{loyal}^{bm}>p_{poach}^{bm}$. Hence, loyal consumers end up paying a higher price than those that are poached by the other firm ($p_{loyal}^{bm}>p_{poach}^{bm}$). Also, the possibility of poaching makes competition particularly fierce in the second period ($p_{loyal}^{bm}<p^h$). In the first period, the effect is ambiguous. On the one hand, consumers are less sensitive to first period prices;[4] marginal consumers know that if they are tempted to consume their less-preferred product, that will imply higher prices in period 2.[5] On the other hand, as switching costs increase, firms are more eager to attract consumers in period 1, as consumers will be less inclined to switch, so second-period profits increase.[6] As a result, first-period prices are higher ($p_1^{bm}>p_h$) with low switching costs, but lower ($p_1^{bm}<p_h$) with high switching costs. We also have:

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Corollary 4.1. *The total discounted price paid by both loyal and non-loyal consumers is lower than that in a standard Hotelling model. All consumers are strictly better off. Firms are worse off. Total welfare decreases.*

Proof of Corollary 4.1. For loyals, the effect on total discounted price is

$$\Delta P_{loyal}=p_1^{bm}+\delta p_{loyal}^{bm}-(1+c)\left(1+\delta\right)=-\frac{1}{3}\bar{z}\delta<0,$$

Finally, firm A can defect by setting a lower p_{AA}^2, such that it serves all the high types. In that case the profit function we use in the main text overestimates true profits (since it assumes a $\hat{x}_{AH}>1/2$ rather than the true $\hat{x}_{AH}=1/2$). As we cannot find a profit-increasing defection when looking at an inflated profit function, such a profit-increasing defection definitely does not exist when looking at the true profit function.

[4]Note from (4.18) that $\partial\hat{x}_1/\partial\left(p_B^1-p_A^1\right)=3/\left(6+2\delta\right)$, whereas in a standard Hotelling model, we would have $\partial\hat{x}/\partial\left(p_B-p_A\right)=1/2$.

[5]From (4.12), an increase in $\hat{x}_1$, the size of segment A, implies that both p_{AA}^2 and p_{BA}^2 increase.

[6]From (4.19), in equilibrium $\partial\Pi_A/\partial\bar{z}=\delta\left(1+2\bar{z}\right)/9>0$.

hence they are better off. For consumers that are poached

$$\Delta P_{poach} = p_1^{bm} + \delta p_{poach}^{bm} - (1+c)(1+\delta) = -\frac{1}{3}\delta(1+3\bar{z}) < 0.$$

These consumers now incur switching costs and a disutility from no longer consuming their preferred product in period 2. However, if they would choose not to switch, they would still be strictly better off than in a Hotelling model. Revealed preference implies that their net utility from switching is only higher. As total discounted prices decrease an the markets is covered, profits are lower. For total welfare, prices are just a transfer. With poaching, some consumers incur switching costs, and a utility loss from no longer consuming their preferred product. From a welfare perspective, that is a loss. ■

4.4 Introducing retention offers

Preliminaries We now consider the full model and analyze whether there is an equilibrium in which retention offers occur. We thus look for an equilibrium where low types that do not switch always pay the retention price while high types that do not switch pay the loyalty price.

We solve with backward induction and again focus on segment A; consumers that have bought from firm A in period 1. For retention offers to occur in equilibrium, we need that low types that buy again from A go for the retention offer p_A^R, while high types prefer the loyalty price p_{AA}^2. For the low types, we thus need that the inspection costs z_L^1 are smaller than the difference between p_A^R and p_{AA}^2, while for the high types the opposite is true. An equilibrium with retention offers thus requires

$$z_L^1 < p_{AA}^2 - p_A^R < z_H^1. \tag{4.21}$$

As we are interested in situations where retention offers indeed occur in equilibrium, below we will derive parameter restrictions such that these conditions are indeed satisfied. Note that in equilibrium all low types incur the inspection costs z_L^1. Hence, a low type that decides to switch rather than stay loyal to firm A, only incurs additional switching costs z_L^2. In equilibrium, the loyal high types do not incur inspection costs. Hence, high types that decide to switch incur an additional z_H. We denote by $\tilde{z}$ the weighted average of these additional switching costs. Hence

$$\tilde{z} \equiv \lambda z_L^2 + (1-\lambda) z_H. \tag{4.22}$$

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Second period, second stage In stage 2 of period 2, firm A sets retention price p_A^R to maximize profits, given the loyalty price p_{AA}^2 and the poaching price p_{BA}^2 that were set in stage 1. All low types have already incurred the preparation costs z_L^1. A low type that switches thus incurs an additional z_L^2 and would pay p_A^R when sticking to A. A high type that switches incurs an additional z_H and would pay p_{AA}^2 when sticking to A. Hence, the indifferent consumers in segment A are given by

$$\hat{x}_{AL}^2 = \tfrac{1}{2}(1 + p_{BA}^2 - p_A^R + z_L^2); \qquad \hat{x}_{AH}^2 = \tfrac{1}{2}(1 + p_{BA}^2 - p_{AA}^2 + z_H). \tag{4.23}$$

Firm A's second-period profits from segment A now equal

$$\begin{aligned}\Pi_{AA}^2 &= \lambda(p_A^R - c)\hat{x}_{AL}^2 + (1-\lambda)(p_{AA}^2 - c)\hat{x}_{AH}^2 \\ &= \frac{1}{2}\lambda(p_A^R - c)\left(1 + p_{BA}^2 - p_A^R + z_L^2\right) \\ &\quad + \frac{1}{2}(1-\lambda)(p_{AA}^2 - c)\left(1 + p_{BA}^2 - p_{AA}^2 + z_H\right).\end{aligned} \tag{4.24}$$

Maximizing with respect to p_A^R yields

$$p_A^R = \frac{1}{2}\left(1 + p_{BA}^2 + z_L^2 + c\right). \tag{4.25}$$

Second period, first stage Maximizing (4.24) with respect to p_{AA}^2 yields the first-stage best-reply function for firm A:

$$p_{AA}^2 = \frac{1}{2}\left(1 + p_{BA}^2 + z_H + c\right). \tag{4.26}$$

Firm B's second-period profits on segment A are given by

$$\Pi_{BA}^2 = \lambda(p_{BA}^2 - c)\left(\hat{x}_L^1 - \hat{x}_{AL}^2\right) + (1-\lambda)(p_{BA}^2 - c)\left(\hat{x}_H^1 - \hat{x}_{AH}^2\right). \tag{4.27}$$

Firm B anticipates that A will set p_A^R according to (4.25). Using (4.23), we can write

$$\begin{aligned}\Pi_{BA}^2 &= \lambda(p_{BA}^2 - c)\left(\hat{x}_L^1 - \frac{1}{4}\left(1 + p_{BA}^2 + z_L^2 - c\right)\right) \\ &\quad + (1-\lambda)(p_{BA}^2 - c)\left(\hat{x}_H^1 - \frac{1}{2}\left(1 + p_{BA}^2 - p_{AA}^2 + z_H\right)\right).\end{aligned}$$

Taking the first-order condition yields the reaction function

$$p_{BA}^2 = \frac{4\hat{x}^1 + 2\,(1-\lambda)\,p_{AA}^2 - \tilde{z} - (1-\lambda)\,z_H + 2c}{4 - 2\lambda} - \frac{1}{2}.$$

For the equilibrium, we plug in the reaction function of firm A, (4.26) to find

$$p_{BA}^2 = \frac{4\hat{x}^1 + (1-\lambda)\left(1+p_{BA}^2\right) - \tilde{z} + (3-\lambda)\,c}{4-2\lambda} - \frac{1}{2}.$$

Hence

$$p_{BA}^2 = c + b, \tag{4.28}$$

with

$$b \equiv \frac{4\hat{x}^1 - \tilde{z} - 1}{3-\lambda} \tag{4.29}$$

the equilibrium price-cost margin on B's poaching prices. From (4.25) and (4.26) we then have

$$p_A^R = c + \tfrac{1}{2}\left(1 + z_L^2 + b\right); \qquad p_{AA}^2 = c + \tfrac{1}{2}\left(1 + z_H + b\right). \tag{4.30}$$

while equilibrium market shares follow directly from (4.23):

$$\hat{x}_{AL}^2 = \tfrac{1}{4}\left(1 + z_L^2 + b\right); \qquad \hat{x}_{AH}^2 = \tfrac{1}{4}\left(1 + z_H + b\right), \tag{4.31}$$

provided that these expressions are strictly between 0 and the relevant $\hat{x}_i^1$. Given that we already impose (4.1), parameter restriction (4.3) assures that that is the case in equilibrium.[7] For equilibrium profits for A, we plug these values into (4.24) to find

$$\Pi_{AA}^2 = \frac{1}{8}\lambda\left(1 + z_L^2 + b\right)^2 + \frac{1}{8}\left(1-\lambda\right)\left(1 + z_H + b\right)^2. \tag{4.32}$$

Similarly, using (4.27), profits for firm B can be shown to equal

$$\Pi_{BA}^2 \;=\; b\left(\hat{x}^1 - \frac{1}{4}\lambda\left(1+z_L^2\right) - \frac{1}{4}\left(1-\lambda\right)\left(1+z_H\right) - \frac{1}{4}b\right)$$

[7] First note that we immediately have $\hat{x}_{Ai}^2 > 0$. To have $\hat{x}_{Ai}^2 < 1/2$, we need

$$\max\left\{z_L^2, z_H\right\} + b < 1.$$

Note

$$b = \frac{1-\tilde{z}}{3-\lambda} = \frac{1-\left(\lambda z_L^2 + (1-\lambda)\,z_H\right)}{3-\lambda}$$

which is increasing in λ (as the numerator is increasing and the denominator decreasing). We want the condition to be satisfied for all λ. It is most restrictive for $\lambda = 1$, so we require

$$\max\left\{z_L^2, z_H\right\} + \frac{1}{2}\left(1 - z_L^2\right) < 1.$$

For the low types, this implies $z_L^2 < 1$, which is always satisfied given that (4.1) is satisfied. For the high types we need $z_H + \frac{1}{2}\left(1 - z_L^2\right) < 1$, which is implied by (4.3).

$$= \frac{1}{4}b\left(4\hat{x}^1 - \tilde{z} - 1 - b\right) = \frac{1}{4}(2-\lambda)b^2. \tag{4.33}$$

On segment B, we have a similar analysis that yields

$$p_{AB}^2 = c + a; \qquad \Pi_{AB}^2 = \tfrac{1}{4}(2-\lambda)a^2, \tag{4.34}$$

with

$$a \equiv \frac{3 - 4\hat{x}^1 - \tilde{z}}{3-\lambda}$$

the price-cost margin on A's poaching prices.

First period Again, the indifferent consumer in period 1 is given by (4.17): retention prices do not affect first-period market shares, as the marginal consumer will always switch. Substituting for p_{AB}^2 and p_{BA}^2 from (4.28) and (4.34) into (4.17):

$$\hat{x}_i^1 = \frac{1 + p_B^1 - p_A^1 - \delta\left(\frac{8\hat{x}^1 - 1 - \lambda}{3-\lambda}\right)}{2 - 2\delta}.$$

Using (4.9), substituting from the above equations and solving for $\hat{x}^1$ yields

$$\hat{x}_L^1 = \hat{x}_H^1 = \hat{x}^1 = \frac{(3-\lambda)\left(1 + p_B^1 - p_A^1\right) + \delta\,(1+\lambda)}{8\delta + (3-\lambda)\,(2-2\delta)}.$$

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Total profits for firm A are now given by

$$\begin{aligned}
\Pi_A &= (p_A^1 - c)\hat{x}^1 + \delta\Pi_{AA}^2 + \delta\Pi_{AB}^2 \\
&= (p_A^1 - c)\hat{x}^1 + \frac{\delta}{8}\lambda\left(b + 1 + z_L^2\right)^2 \\
&\quad + \frac{\delta}{8}\,(1-\lambda)\,(b + 1 + z_H)^2 + \frac{\delta}{4}(2-\lambda)a^2.
\end{aligned} \tag{4.35}$$

Taking the derivative with respect to p_A^1 :

$$\begin{aligned}
\frac{\partial \Pi_A}{\partial p_A^1} &= \hat{x}^1 + (p_A^1 - c)\frac{\partial \hat{x}^1}{\partial p_A^1} + \frac{\delta\lambda}{4}\left(b + 1 + z_L^2\right)\frac{\partial b}{\partial \hat{x}^1}\frac{\partial \hat{x}^1}{\partial p_A^1} \\
&\quad + \frac{\delta}{4}\,(1-\lambda)\,(b + 1 + z_H)\frac{\partial b}{\partial \hat{x}^1}\frac{\partial \hat{x}^1}{\partial p_A^1} + \frac{\delta a}{2}(2-\lambda)\frac{\partial a}{\partial \hat{x}^1}\frac{\partial \hat{x}^1}{\partial p_A^1} \\
&= \hat{x}^1 + (p_A^1 - c)\frac{\partial \hat{x}^1}{\partial p_A^1} + \frac{\delta}{4}\,(b + 1 + \tilde{z})\frac{\partial b}{\partial \hat{x}^1}\frac{\partial \hat{x}^1}{\partial p_A^1} + \frac{\delta a}{2}(2-\lambda)\frac{\partial a}{\partial \hat{x}^1}\frac{\partial \hat{x}^1}{\partial p_A^1}.
\end{aligned}$$

With

$$\frac{\partial b}{\partial \hat{x}^1} = -\frac{\partial a}{\partial \hat{x}^1} = \frac{4}{3-\lambda}$$

$$\frac{\partial \hat{x}^1}{\partial p_A^1} = \frac{-(3-\lambda)}{8\delta + (3-\lambda)(2-2\delta)},$$

the first-order condition becomes[8]

$$\hat{x}^1 - \frac{(3-\lambda)\left(p_A^1 - c\right) + (b+1+\tilde{z})\,\delta - 2(2-\lambda)\,a\delta}{8\delta + (3-\lambda)(2-2\delta)} = 0.$$

Equilibrium requires $p_A^1 = p_B^1$, hence $\hat{x} = 1/2$ and $a = b = \frac{1-\tilde{z}}{3-\lambda}$. Solving for equilibrium prices then yields:

Proposition 4.2. *With the possibility of retention offers, equilibrium first-period, loyalty, poaching and retention prices are given by*

$$\begin{aligned} p_1^{ret} &= c + 1 - 3\delta\frac{\tilde{z}(2-\lambda)}{(3-\lambda)^2} + \delta\frac{\left(3-\lambda^2+\lambda\right)}{(3-\lambda)^2} \\ p_{loyal}^{ret} &= c + \frac{1}{2}(1 + z_H + b)\,; \\ p_{poach}^{ret} &= c + b; \\ p_{retent}^{ret} &= c + \frac{1}{2}\left(1 + z_L^2 + b\right), \end{aligned} \tag{4.36}$$

with $b = (1-\tilde{z})/(3-\lambda)$. *Equilibrium profits are given by*

$$\Pi_A^{ret} = \frac{1}{2}\left(p^1 - c\right) + \frac{\delta}{8}\lambda\left(b+1+z_L^2\right)^2 + \frac{\delta}{8}(1-\lambda)(b+1+z_H)^2 + \frac{\delta}{4}(2-\lambda)b^2. \tag{4.37}$$

In this case, we have $p^h > p_{loyal}^{ret} > p_{retent}^{ret} > p_{poach}^{ret}$. Again, the comparison of p_1^{ret} and p_h is ambiguous.[9]

Consumers that go for a retention offer thus pay a *higher* price than what they would pay if they would switch. As their original supplier knows that these consumers have a

[8] It is readily checked that the second-order condition is satisfied as well.

[9] Clearly $p_{loyal}^{ret} > p_{retent}^{ret}$. As $b = \frac{1-\tilde{z}}{3-\lambda} < \frac{1}{2}$ we have that $p_{poach}^{ret} < p_{retent}^{ret}$. Note

$$p_{loyal}^{ret} - p_h = \frac{1}{2}\left(\frac{\lambda\left(1-z_L^2\right) - 2(1-z_H)}{3-\lambda}\right)$$

The numerator is given by

$$\begin{aligned} \lambda - \lambda z_L^2 + 2z_H - 2 &< \lambda - \lambda(2z_H - 1) + 2z_H - 2 \\ &= -2(1-z_H)(1-\lambda) < 0 \end{aligned}$$

where the first inequality follows from (4.3). Hence $p_{loyal}^{ret} < p_h$. This establishes the ranking. Finally, note that

$$p_1^{ret} - p_h = -3\delta\frac{\tilde{z}(2-\lambda)}{(3-\lambda)^2} + \delta\frac{\left(3-\lambda^2+\lambda\right)}{(3-\lambda)^2}.$$

With $\lambda = 0$, this equals $\delta(1-2z_H)/3$, the sign of which is ambiguous.

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preference for their product, they do not have to fully compensate for the lower price of the other firm.

Poaching prices are decreasing in switching costs z_H and z_L^2: the higher these, the more of an effort firms have to make to poach consumers. At the same time, an increase in these switching costs increases loyalty prices, as firms have to make less of an effort to retain consumers. These comparative statics are the same to those in the benchmark model. Retention prices increase in z_L^2. Only low types end up paying this price, and an increase in their switching costs makes it easier to retain them. First period prices decrease in z_H and z_L^2. As it becomes harder to poach consumers in the second period, it becomes more profitable to attract them in the first period. Hence, an increase in switching costs deceases first-period prices.

In the benchmark model (that added switching costs and poaching to a standard Hotelling model), we compared the total discounted price, consumer welfare, profits and total welfare to that in the Hotelling model. It is less straightforward to do that once we also add retention offers; all comparisons then become ambiguous.[10] Anyhow, it is far more interesting to compare a world in which retention offers are possible to one where they are not; doing so allows us to truly evaluate the welfare effects of retention offers *per se*. We wil do so in the next section.

As a final technical aside, note that we need some parameter restrictions for our separating condition (4.21) to be satisfied. From (4.30), this requires

$$z_H^1 + z_H^2 > 2z_L^1 + z_L^2; \tag{4.38}$$

$$z_H^2 - z_H^1 < z_L^2. \tag{4.39}$$

4.5 The effects of retention offers

Comparing the model with the possibility of retention offers to the benchmark model, we now have the following:

Proposition 4.3. *Introducing the possibility of retention offers in our benchmark model has the following effects on prices:*

(i) First-period prices, poaching prices, and loyalty prices all increase.

[10] See Appendix A for details.

(ii) *New poaching prices are still lower than benchmark loyalty prices. New loyalty prices are higher than benchmark poaching prices.*

(iii) *Retention prices are always higher than benchmark poaching prices, and higher than benchmark loyalty prices if and only if λ is high enough.*

Summarizing, we have $p_1^{ret} > p_1^{bm}$, while the effects on second-period prices are as follows:

	p_{poach}^{bm}	p_{loyal}^{bm}
p_{poach}^{ret}	$>$	$<$
p_{retent}^{ret}	$>$	$\gtrless$
p_{loyal}^{ret}	$>$	$>$

Proof. In appendix B. ■

To see what drives these results, note the following. First, for the low types, effective switching costs decrease. *Ceteris paribus*, when viewed in isolation, this would lead to a lower price charged by firm A and a higher price charged by firm B (see e.g. equation (4.12)). Second, the loyalty price will now only be paid by high types. These are reluctant to switch, leading to higher loyalty prices. In turn, when viewed in isolation, these higher loyalty prices also allow firm B to charge higher poaching prices. Both channels lead to higher poaching prices. Also, note that average effective switching costs decrease. That implies that firms become less eager to capture consumers in period 1, hence first-period prices increase.

The welfare effects of retention offers are as follows:

Proposition 4.4. *The possibility of retention offers increases equilibrium profits and decreases total consumer surplus. Each high type consumer is worse off. Only some individual low type consumers that end up paying the retention price, may be better off. Total welfare effects are ambiguous.*

Proof. In appendix B. ■

Hence, although the possibility of retention offers may seem at first sight to benefit consumers, that is not the case in our equilibrium analysis. In our model, retention offers serve to screen consumers with high switching costs from those with low switching costs, allowing firms to effectively price discriminate against high cost consumers, which hurts such consumers. Yet, consumers with low switching costs are often also worse off. They are forced to incur some of their switching costs in order to qualify for the retention offer, even if they do not intend to switch. Moreover, this lowers their effective switching costs, making competition for them less fierce in the first period. As a result, firms benefit from having the possibility of making retention offers.

4.6 Conclusion

In this chapter, we studied the practice of retention offers. In a two-period Hotelling model, two firms practice behavior based price discrimination. In the second period, they can try to poach consumers by offering them a better deal. However, firms can retaliate by making a retention offer. Consumers differ in their switching costs. In equilibrium, low-switching-cost consumers always solicit a retention offer, while this is too costly for high-switching-cost consumers. As a result, retention offers allow firms to effectively price discriminate against high-switching-cost consumers.

We find that the possibility of retention offers increases firm profits. All high-cost consumers are worse off, but some low-cost consumers may benefit. Prices increase. From a welfare perspective, more wasteful switching costs are incurred, as all low-cost consumers solicit a costly offer from the competitor in order to secure a retention price.

4.A Comparing retention offers to prices in a Hotelling model

Corollary 4.2. *In a model with retention offers, compared to a Hotelling model,*

(i) the total discounted price paid by loyal consumers is lower for low enough λ and the comparison is ambiguous otherwise;

(ii) that paid by consumers that switch is either always lower, or is higher for high enough λ and lower otherwise;

(iii) that paid by consumers that pay the retention price is higher for high enough λ and lower otherwise;

(iv) firms are worse off for low enough λ, and the comparison is ambiguous otherwise;

(v) total welfare decreases.

Proof. First consider the loyal consumers. Using (4.22) and (4.36),

$$\begin{aligned}
P_{loyal}^{ret} &\equiv p_1^{ret} + \delta p_{loyal}^{ret} \\
&= c + 1 - 3\delta \frac{\tilde{z}\,(2-\lambda)}{(3-\lambda)^2} + \delta \frac{\left(3-\lambda^2+\lambda\right)}{(3-\lambda)^2} + \delta\left(c + \frac{1}{2}\,(1 + z_H + b)\right) \\
&= P^h - 3\delta \frac{\tilde{z}\,(2-\lambda)}{(3-\lambda)^2} + \delta \frac{\left(3-\lambda^2+\lambda\right)}{(3-\lambda)^2} + \delta\left(\frac{1}{2}\,(z_H + b) - \frac{1}{2}\right) \\
&= P^h + \frac{1}{2}\delta \frac{\lambda\left(16z_H - 15z_L^2 + 7\right) - \lambda^2\left(6z_H - 7z_L^2 + 3\right) - 6z_H}{(3-\lambda)^2}.
\end{aligned}$$

With $\lambda = 0$, the numerator is $-6z_H < 0$. With $\lambda = 1$, it is $4z_H - 8z_L^2 + 4 > 0$, which is ambiguous. For consumers that are poached, we have from (4.22) and (4.36)

$$\begin{aligned}
P_{poach}^{ret} &\equiv p_1^{ret} + \delta p_{poach}^{ret} \\
&= P^h - 3\delta \frac{\tilde{z}\,(2-\lambda)}{(3-\lambda)^2} + \delta \frac{\left(3-\lambda^2+\lambda\right)}{(3-\lambda)^2} + \delta\left(\frac{1-\tilde{z}}{3-\lambda} - 1\right) \\
&= P^h + \delta \frac{\tilde{z}\,(4\lambda - 9) + 6\lambda - 2\lambda^2 - 3}{(3-\lambda)^2}.
\end{aligned}$$

For $\lambda = 0$, the numerator is $-9\tilde{z} - 3 < 0$. For $\lambda = 1$, it is $1 - 5z_L^2$, which has ambiguous sign. The derivative of the numerator with respect to λ is $4\tilde{z} + 6 - 4\lambda > 0$.

For consumers that pay the retention price, again using (4.22) and (4.36),

$$\begin{aligned}
P_{retent}^{ret} &= p_1^{ret} + \delta p_{retent}^{ret} \\
&= P^h - 3\delta \frac{\tilde{z}\,(2-\lambda)}{(3-\lambda)^2} + \delta \frac{\left(3-\lambda^2+\lambda\right)}{(3-\lambda)^2} + \delta\left(\frac{1}{2}\left(z_L^2 + \frac{1-\tilde{z}}{3-\lambda}\right) - \frac{1}{2}\right) \\
&= P^h + \frac{1}{2}\delta \frac{\lambda\left(22z_H - 21z_L^2 + 7\right) - \lambda^2\left(7z_H - 8z_L^2 + 3\right) + 9z_L^2 - 15z_H}{(3-\lambda)^2}.
\end{aligned}$$

For $\lambda = 0$, the numerator is $9z_L^2 - 15z_H < 0$. For $\lambda = 1$ it is $4 - 4z_L^2 > 0$. The derivative with respect to λ is

$$z_H + 2\lambda z_L^2 + 7\,(3 - 2\lambda)\left(z_H - z_L^2\right) + 7 - 6\lambda > 0.$$

Profits in a standard Hotelling model are $\Pi^h = \frac{1}{2}\,(1+\delta)$. We thus have

$$\begin{aligned}
\Pi^{ret} - \Pi^h &= \frac{1}{2}\left(-3\delta\tilde{z}\frac{(2-\lambda)}{(3-\lambda)^2} + \frac{\delta(3-\lambda^2+\lambda)}{(3-\lambda)^2}\right) + \frac{\delta}{8}\lambda\left(b + 1 + z_L^2\right)^2 \\
&\quad + \frac{\delta}{8}\,(1-\lambda)\,(b + 1 + z_H)^2 + \frac{\delta}{4}\,(2-\lambda)\,b^2 - \frac{1}{2}\delta.
\end{aligned}$$

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With $\lambda = 0$, we have $\tilde{z} = z_H$ and $b = (1 - z_H)/3$, so this expression simplifies to $\delta\left(2(z_H)^2 - 4z_H - 1\right)/18 < 0$. With $\lambda = 1$, we have $\tilde{z} = z_L^2$ and $b = \left(1 - z_L^2\right)/2$, so the expression simplifies to $\left(3(z_L^2)^2 - 10z_L^2 + 7\right)/32$, which has ambiguous sign.

This establishes the result on profits. For total welfare, note that prices are just a transfer. With retention offers, however, some consumers incur switching costs, while no longer consuming their preferred product. From a welfare perspective, that is a loss. ■

4.B Proofs

Proof of Proposition 4.3. To prove the Proposition, we will first establish that $p_1^{ret} > p_1^{bm}$ and then go through all cells in the table.

$\mathbf{p}_1^{ret} > \mathbf{p}_1^{bm}$:

Consider the difference between p_1^{ret} and p_1^{bm} :

$$\begin{aligned}
\Delta p^1 \equiv\ & p_1^{ret} - p_1^{bm} = \\
& -3\delta\frac{\tilde{z}\,(2-\lambda)}{(3-\lambda)^2} + \delta\frac{\left(3-\lambda^2+\lambda\right)}{(3-\lambda)^2} - \frac{\delta}{3}\,(1-2\bar{z}) \\
=\ & \lambda\delta\frac{6z_L^1\,(3-2\lambda) - z_H\,(3-5\lambda) - 4\lambda - 2\lambda^2\,(z_H - z_L) + 9 - 3\lambda z_L^2}{3\,(3-\lambda)^2} \\
>\ & \lambda\delta\frac{6z_L^1\,(3-2\lambda) - (3-5\lambda) - 4\lambda - 2\lambda^2\,(z_H - z_L) + 9 - 3\lambda}{3\,(3-\lambda)^2} \\
=\ & \lambda\delta\frac{6z_L^1\,(3-2\lambda) + 6 + 2\lambda - 2\lambda^2\,(z_H - z_L)}{3\,(3-\lambda)^2} > 0.
\end{aligned}$$

This establishes the result.

$\mathbf{p}_{poach}^{ret} > \mathbf{p}_{poach}^{bm}$:

$$\begin{aligned}
p_{poach}^{ret} - p_{poach}^{bm} &= \frac{1-\tilde{z}}{3-\lambda} - \frac{1}{3}(1-\bar{z}) \\
&> \frac{1}{3}\,(1-\tilde{z}) - \frac{1}{3}(1-\bar{z}) = \frac{1}{3}\lambda z_L^1 > 0,
\end{aligned}$$

where the first inequality follows from $\lambda > 0$. This establishes the result.

$\mathbf{p}_{loyal}^{ret} > \mathbf{p}_{loyal}^{bm}$:

$$\begin{aligned}\Delta p_{\text{loyal}} &\equiv p_{loyal}^{ret} - p_{loyal}^{bm} = \frac{1}{2}\left(1 + z_H + \frac{1-\tilde{z}}{3-\lambda}\right) - \frac{1}{3}(2 + \tilde{z} + \lambda z_L^1) \\ &> \frac{1}{2}\left(1 + z_H + \frac{1-\tilde{z}}{3}\right) - \frac{1}{3}(2 + \tilde{z} + \lambda z_L^1) = \frac{1}{2}z_H - \frac{1}{2}\tilde{z} - \frac{1}{3}\lambda z_L^1 \\ &= \frac{1}{6}\lambda\left(3z_H - 2z_L^1 - 3z_L^2\right) = \frac{1}{6}\lambda\left(3z_H - 3z_L + z_L^1\right) > 0\end{aligned}$$

as $z_H > z_L$. This establishes the result.

$\mathbf{p}_{poach}^{ret} < \mathbf{p}_{loyal}^{bm}$:

$$p_{\text{poach}}^{\text{ret}} - p_{\text{loyal}}^{\text{bm}} = \frac{1-\tilde{z}}{3-\lambda} - \frac{1}{3}(2 + \bar{z}) < 0,$$

as the first term is strictly smaller than 1/2, while the second term is strictly bigger than 2/3. This establishes the result.

$\mathbf{p}_{retent}^{ret} > \mathbf{p}_{poach}^{bm}$:
Above, we showed that $p_{poach}^{ret} > p_{poach}^{bm}$. From section 4.4, $p_{retent}^{ret} > p_{poach}^{ret}$. This establishes the result.

$\mathbf{p}_{retent}^{ret} \gtrless \mathbf{p}_{loyal}^{bm}$:

$$\begin{aligned}p_{\text{retent}}^{ret} - p_{\text{loyal}}^{\text{bm}} &= \frac{1}{2}\left(1 + z_L^2 + \frac{1-\tilde{z}}{3-\lambda}\right) - \frac{1}{3}(2 + \bar{z}) \\ &= \frac{\lambda\left(1 + 11z_H - 6z_L^1 - 12z_L^2\right) - 9\left(z_H - z_L^2\right) - 2\lambda^2\left(z_H - z_L\right)}{6\left(3-\lambda\right)}.\end{aligned}$$

This expression is positive if and only if the numerator is positive. For $\lambda = 0$, it equals $-9\left(z_H - z_L^2\right) < 0$. For $\lambda = 1$, it equals $2z_L - 6z_L^1 - 3z_L^2 + 1 = 1 - z_L^2 - 4z_L^1$, of which the sign is ambiguous. This establishes the result.

$\mathbf{p}_{loyal}^{ret} > \mathbf{p}_{poach}^{bm}$:

$$p_{\text{loyal}}^{\text{ret}} - p_{\text{poach}}^{\text{bm}} = \frac{1}{2}\left(1 + z_H + \frac{1-\tilde{z}}{3-\lambda}\right) - \frac{1}{3}(1 - \bar{z}) > 0,$$

as the first term is strictly larger than 1/2, while the second is strictly smaller than 1/3. This establishes the result. ■

Proof of Proposition 4.4

We now set about proving Proposition 4.4. We proceed as follows. First, we compare the total discounted prices that consumers end up paying under different circumstances. These comparisons will prove useful in deriving our results. We then consider how individual consumers are affected, and look at total welfare. After that, we consider firm profits and total welfare, respectively.

The effect on total discounted prices

We can establish the following:

Lemma 4. *Introducing the possibility of retention offers often increases the total discounted prices paid by consumers. The only exceptions are the case in which a consumer would be loyal in the benchmark, but would either get poached or get a retention offer when retention offers can be made. Such consumers pay a lower total discounted price if λ is low enough, but may pay a higher total discounted price if λ is high enough. Summarizing, the effects are as follows:*

	P^{bm}_{poach}	P^{bm}_{loyal}
P^{ret}_{poach}	$>$	$\gtrless$
P^{ret}_{retent}	$>$	$\gtrless$
P^{ret}_{loyal}	$>$	$>$

Proof. Results involving $P^{\mathrm{bm}}_{\mathrm{poach}}$ follow directly from Proposition 4.3, as does $P^{\mathrm{ret}}_{\mathrm{loyal}} > P^{\mathrm{bm}}_{\mathrm{loyal}}$. Let us now consider the expression $\Delta P_{\text{rr-bl}} \equiv P^{\mathrm{ret}}_{\mathrm{retent}} - P^{\mathrm{bm}}_{\mathrm{loyal}}$. At $\lambda = 0$, we have that $p^{\mathrm{ret}}_1 = p^{\mathrm{bm}}_1$, hence $\Delta P_{\text{rr-bl}} = p^{\mathrm{ret}}_{\mathrm{retent}} - p^{\mathrm{loyal}}_{\mathrm{bm}} = \delta \frac{z_L^2 - z_H}{2} < 0$. For $\lambda = 1$, we have $\Delta P_{\text{rr-bl}} = \delta \left(z_L + 3z_L^1 - 3z_L^2 + 6 \right) / 12 > 0$, which implies the statement in the Lemma concerning this case. Finally, consider the expression $\Delta P_{\text{rp-bl}} \equiv P^{\mathrm{ret}}_{\mathrm{poach}} - P^{\mathrm{bm}}_{\mathrm{loyal}}$. It can be shown that for $\lambda = 0$, we have that $\Delta P_{\text{rp-bl}} = -\delta \left(2z_H + 1 \right) / 3 < 0$, while for $\lambda = 1$, we have $\Delta P_{\text{rp-bl}} = \frac{1}{12} \delta \left(4z_L^1 - 11z_L^2 + 3 \right)$, the sign of which is ambiguous. ■

The effect on consumer welfare

Lemma 5. *The possibility of retention offers makes all consumers strictly worse off, apart possibly from those that pay the retention price in period 2.*

Proof. For a single consumer, there are 6 possible options: she is poached both in the benchmark as well as in the scenario with retention offers; she is loyal in both cases, she is poached in the benchmark and loyal in the retention scenario; she is loyal in the benchmark but poached in the retention scenario; she is loyal in the benchmark, but pays a retention price in the retention scenario, or she is poached in the benchmark and pays a retention price in the retention scenario. We will refer to these 6 options as PP, LL, PL, LP, LR and PR respectively. Note that not all 6 options necessarily occur in equilibrium, depending on parameter values, either one may occur.

In all cases, the total discounted price that a consumer ends up paying is a disutility for that consumer. A consumer that is poached in the second period has an additional disutility of, first, the switching costs that she has to incur and, second, the utility mismatch that is caused by the fact that she does no longer consume her preferred product. A consumer that pays the retention price in the second period has an additional disutility that consist of the additional costs she has to incur to prepare for a switch.

A consumer is worse off with the possibility of retention offers if the total disutility she ends up with then is higher than her total disutility in the benchmark. We will refer to the total disutility in scenario x if a consumer ends up paying a price of type y as D_y^x. Going through all possibilities:

4

PP The net difference in disutility in both scenarios equals that in total discounted prices. As $P_{\text{poach}}^{\text{ret}} > P_{\text{poach}}^{\text{bm}}$, we thus have $D_{\text{poach}}^{\text{bm}} < D_{\text{poach}}^{\text{ret}}$.

LL The net difference in disutility in both scenarios equals that in total discounted prices. As $P_{\text{loyal}}^{\text{ret}} > P_{\text{loyal}}^{\text{bm}}$, we thus have $D_{\text{loyal}}^{\text{bm}} < D_{\text{loyal}}^{\text{ret}}$.

PL As this consumer chooses the poaching price in the benchmark, she has $D_{\text{poach}}^{\text{bm}} < D_{\text{loyal}}^{\text{bm}}$. With $D_{\text{loyal}}^{\text{bm}} < D_{\text{loyal}}^{\text{ret}}$, this implies $D_{\text{poach}}^{\text{bm}} < D_{\text{loyal}}^{\text{ret}}$.

LP As this consumer chooses the loyalty price in the benchmark, she has $D_{\text{loyal}}^{\text{bm}} < D_{\text{poach}}^{\text{bm}}$. With $D_{\text{poach}}^{\text{bm}} < D_{\text{poach}}^{\text{ret}}$, this implies $D_{\text{loyal}}^{\text{bm}} < D_{\text{poach}}^{\text{ret}}$.

LR In this case, consider $\Delta D_{\text{rr-bl}} \equiv D_{\text{retent}}^{\text{ret}} - D_{\text{loyal}}^{\text{bm}} = P_{\text{retent}}^{\text{ret}} + \delta z_L^1 - P_{\text{loyal}}^{\text{bm}}$. From the proof of Lemma 4, with $\lambda = 0$, we have that $P_{\text{retent}}^{\text{ret}} - P_{\text{loyal}}^{\text{bm}} = \delta \left(z_L^2 - z_H \right)/2$, hence $\Delta D_{\text{rr-bl}} = \delta \left(z_L^2 + 2z_L^1 - z_H \right)/2 < 0$. With $\lambda = 1$, we have that $P_{\text{retent}}^{\text{ret}} - P_{\text{loyal}}^{\text{bm}} > 0$, hence $\Delta D_{\text{rr-bl}} > 0$, rendering the net effect ambiguous.

PR In this case, consider $\Delta D_{\text{rr-bp}}(x) \equiv D^{\text{ret}}_{\text{retent}} - D^{\text{bm}}_{\text{poach}}(x)$. As we will see below, it is now important to take into account that the disutility of a consumer that is poached depends on her location, which we denote x. We thus have

$$\begin{aligned}\Delta D_{\text{rr-bp}}(x) &= P^{\text{ret}}_{\text{retent}} + \delta z^F_L - P^{\text{bm}}_{\text{poach}} - \delta z_L - \delta m(x) \\ &= P^{\text{ret}}_{\text{retent}} - P^{\text{bm}}_{\text{poach}} - \delta z^2_L - \delta m(x),\end{aligned}$$

with $m(x)$ the mismatch of a consumer located at $x \leq 1/2$ that gets poached: this consumer's transportation costs are now $1 - x$ whereas they would have been x if she consumed her preferred product. Hence $m(x) = 1 - 2x$. From the proof of Proposition 4.3

$$p_1^{ret} - p_1^{bm} = -3\delta\tfrac{\bar{z}(2-\lambda)}{(3-\lambda)^2} + \delta\tfrac{\left(3-\lambda^2+\lambda\right)}{(3-\lambda)^2} - \tfrac{\delta}{3}\left(1 - 2\bar{z}\right)$$

Using the expressions for $p^{\text{bm}}_{\text{loyal}}$ and p^{ret}_{retent} in Propositions 4.1 and 4.2 we find for $\lambda = 0$ $\Delta D_{\text{rr-bp}}(x) = \delta\frac{-4+z_H-3z^2_L+12x}{6}$, which is ambiguous. Similarly, for $\lambda = 1$, $\Delta D_{\text{rr-bp}}(x) = \delta\frac{-2+12z^1_L-6z^2_L+24x}{12}$ which is ambiguous as well.

■

Lemma 6. *Total consumer welfare decreases with the possibility of retention offers. This holds both for the high types as well as for the low types when λ is large enough.*

Proof. For the high types, this follows directly from Lemma 5 (note that high types never pay the retention price). The analysis for the low types is more involved. Consider segment A in the benchmark scenario. Total disutility of the low types that are loyal is given by $\hat{x}^{\text{bm}}_{AL} \cdot D^{\text{bm}}_{\text{loyal}} = \hat{x}^{\text{bm}}_{AL} \cdot P^{\text{bm}}_{\text{loyal}}$. Total disutility of the low types that are poached first consists of $\left(\frac{1}{2} - \hat{x}^{\text{bm}}_{AL}\right)\left(P^{\text{bm}}_{\text{poach}} + \delta z_L\right)$, as these consumers pay $P^{\text{bm}}_{\text{poach}}$ and also incur switching costs in the second period. Moreover, each of these consumers incurs a mismatch: her transportation costs are now $1 - x$ whereas they would have been x if she consumed her preferred product. Hence $m(x) = 1 - 2x$, and the total size of this mismatch equals

$$M^{\text{bm}}_{AL} = \int_{\hat{x}^{\text{bm}}_{AL}}^{1/2} m(x)dx = \left(\frac{1}{2} - \hat{x}^{\text{bm}}_{AL}\right)^2.$$

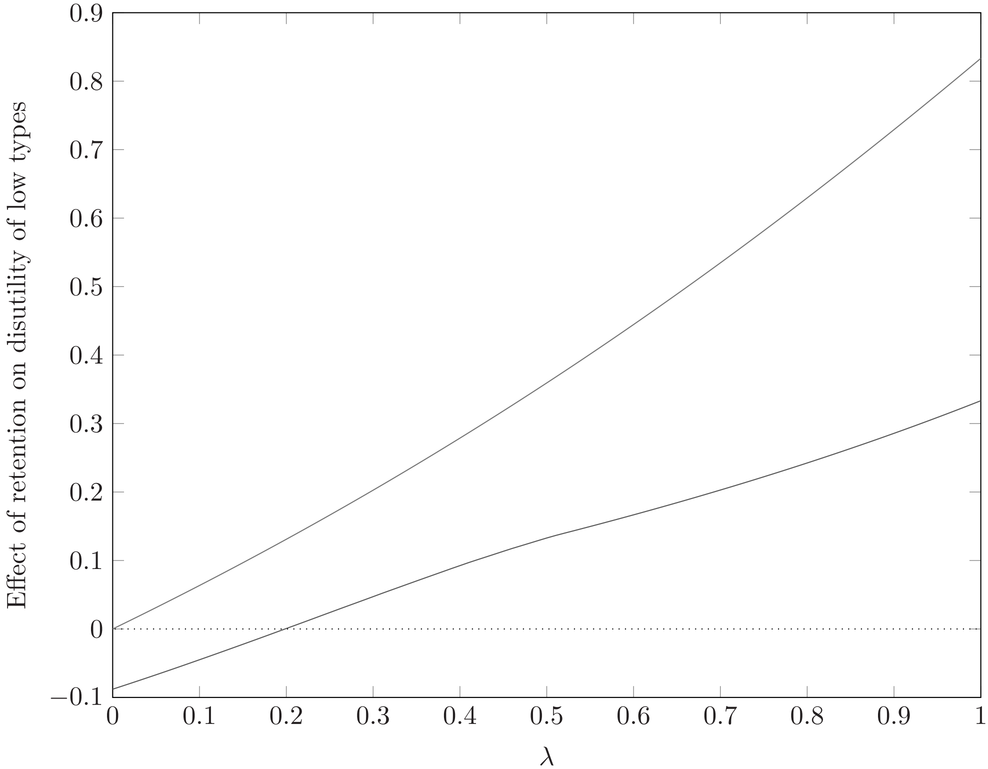
Hence total disutility of the low types in the benchmark is given by

$$D_L^{\mathrm{bm}} \equiv 2\hat{x}_{AL}^{\mathrm{bm}} \cdot P_{\mathrm{loyal}}^{\mathrm{bm}} + 2\left(\frac{1}{2} - \hat{x}_{AL}^{\mathrm{bm}}\right)\left(P_{\mathrm{poach}}^{\mathrm{bm}} + \delta z_L\right) + 2\left(\frac{1}{2} - \hat{x}_{AL}^{\mathrm{bm}}\right)^2.$$

With the possibility of retention offers low type consumers can obtain a retention offer against cost z_L^1. Therefore we find that the total disutility of low types in with the possibility of retention offers is given by

$$D_L^{\mathrm{ret}} \equiv 2\hat{x}_{AL}^{\mathrm{ret}} \cdot \left(P_{\mathrm{retent}}^{\mathrm{ret}} + z_L^1\right) + 2\left(\frac{1}{2} - \hat{x}_{AL}^{\mathrm{ret}}\right)\left(P_{\mathrm{poach}}^{\mathrm{ret}} + \delta z_L\right) + 2\left(\frac{1}{2} - \hat{x}_{AL}^{\mathrm{ret}}\right)^2.$$

Figure 4.2: Effect on disutility of low types of the possibility of retention offers.



The figure gives the upper and lower bound of the effect of the possibility of retention offers on total disutility of the low types, as a function of lambda.

It turns out to be impossible to compare these two expressions analytically. We therefore resort to a numerical analysis. For all values of λ, Figure 3.2 gives the upper and the lower bound on the net welfare effects for the low types of the possibility of having retention offers (thus on $D_L^{\text{ret}} - D_L^{\text{bm}}$ as defined above), for all admissible values of z_L^1, z_L^2, z_H^1, and z_H^2.[11]

From the figure, we have that welfare of the low types may improve with retention offers for low enough λ. Only for those λ, we saw that the low types that buy from A in both scenarios do pay a lower price under retention, while the number of low types that gets poached decreases, lowering their costs of mismatch. For low λ, these positive effects outweigh the negative effects of a higher poaching price and costs to secure a competing offer. Note that we have not weighted the loss by the number of low type consumers, which makes the graph easier to read.

Figure 3.3 shows the change in total consumer welfare. From the figure, for low λ, the positive effects on the low types is outweighted by the negative effect on the high types, rendering the net effect negative. For higher values, the effect is negative for both types, so the total effect also is. ■

The effect on profits

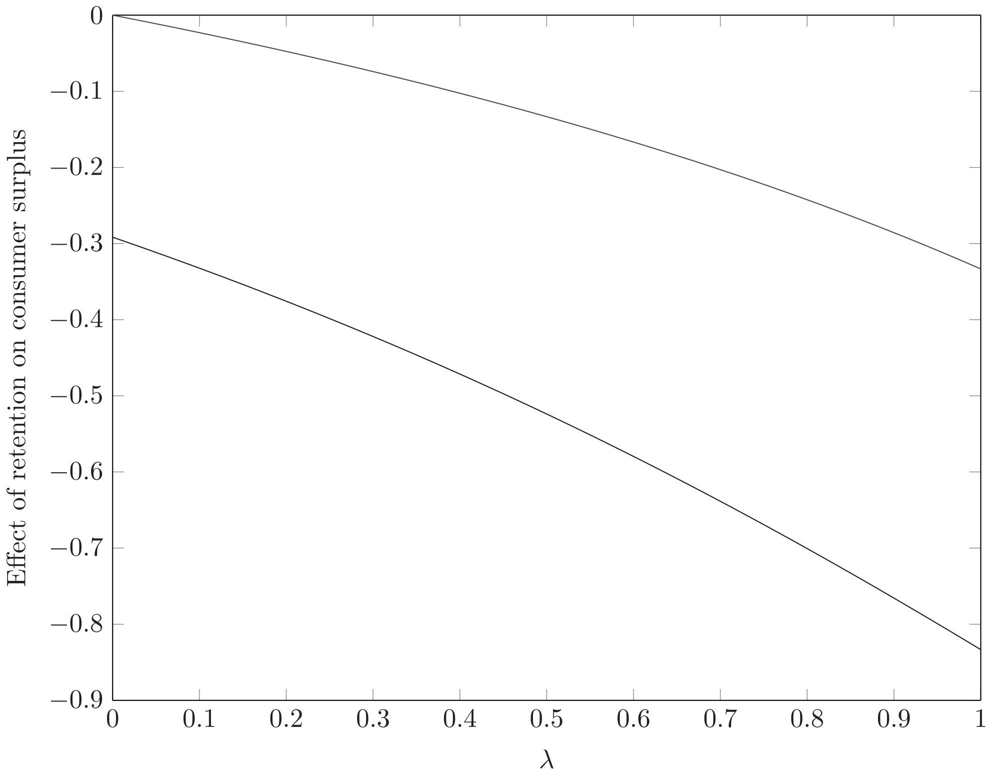
First note that equilibrium profits would obviously increase if all total discounted prices in the retention scenario would be higher than those in the benchmark. Unfortunately, that is not the case. From Lemma 4, consumers may end up paying a lower price if they are loyal in the benchmark, but are either poached or pay the retention price in the case where retention offers are possible.

With a unit mass of consumers that always buy in equilibrium, finding the scenario with the highest profit is equivalent to finding the scenario with the highest average price. Focusing on segment A without loss of generality, we have that the average price paid in the benchmark is given by

$$\begin{aligned} \bar{P}^{\text{bm}} \equiv\ & 2\lambda\hat{x}_{AL}^{\text{bm}} \cdot P_{\text{loyal}}^{\text{bm}} + 2\lambda\left(\frac{1}{2} - \hat{x}_{AL}^{\text{bm}}\right) P_{\text{poach}}^{\text{bm}} \\ & + 2\left(1-\lambda\right)\hat{x}_{AH}^{\text{bm}} \cdot P_{\text{loyal}}^{\text{bm}} + 2\left(1-\lambda\right)\left(\frac{1}{2} - \hat{x}_{AH}^{\text{bm}}\right) P_{\text{poach}}^{\text{bm}}. \end{aligned}$$

[11] The analysis was done in MATLAB. For each of 100 values of λ between 0 and 1, we considered 50 values of z_L^1, z_L^2, z_H^1, as well as z_H^2 to find the highest and the lowest possible value of the price effect of retention offers, taking into account the conditions that have to be satisfied by our switching cost parameters (thus: $z_L^1 < z_H^1$; $z_L^2 < z_H^2$, and conditions (4.1)–(4.3), (4.38) and (4.39)). The analysis took 21 minutes on a 3.30 GHz 4GB RAM Windows 7 PC. The MATLAB code is available upon request. Looking at a finer grid did not appreciably affect the outcomes. Note that in all figures, we have taken $\delta = 1$. The size of δ does not affect the qualitative analysis, however, as all comparisons we consider are proportional to δ.

Figure 4.3: Effect on consumer surplus of the possibility of retention offers.



The figure gives the upper and lower bound of the effect of the possibility of retention offers on total consumer surplus, as a function of lambda.

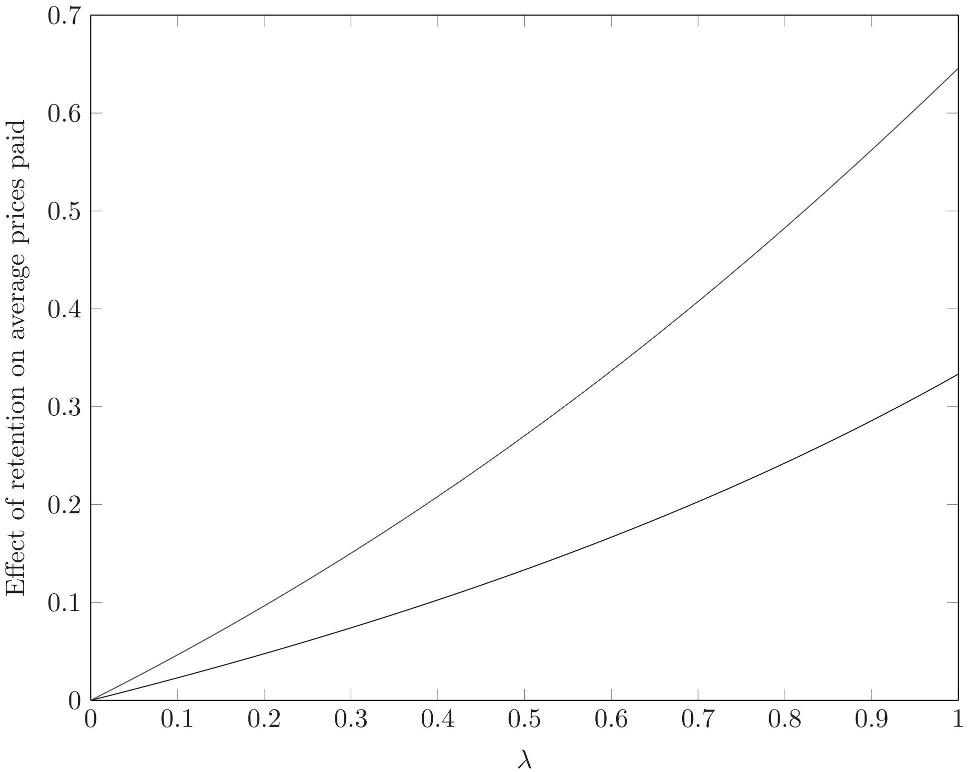
With the possibility of retention, it is given by

$$\begin{aligned}\bar{P}^{\text{ret}} \equiv\ & 2\lambda\hat{x}^{\text{ret}}_{AL} \cdot P^{\text{ret}}_{\text{retent}} + 2\lambda\left(\frac{1}{2} - \hat{x}^{\text{ret}}_{AL}\right) P^{\text{ret}}_{\text{poach}} \\ & +2\left(1-\lambda\right)\hat{x}^{\text{ret}}_{AH} \cdot P^{\text{ret}}_{\text{loyal}} + 2\left(1-\lambda\right)\left(\frac{1}{2} - \hat{x}^{\text{ret}}_{AH}\right) P^{\text{ret}}_{\text{poach}}.\end{aligned}$$

It turns out to be impossible to compare these two expressions analytically. We therefore resort to a numerical analysis. For all values of λ, Figure 3.3 gives the upper and the lower bound on the price effect of the possibility of having retention offers (thus on $\bar{P}^{\text{ret}} - \bar{P}^{\text{bm}}$, as defined above), for all admissible values of the parameters z^1_L, z^2_L, z^1_H, and z^2_H, using an analysis very similar to that described above.

From the figure, it is immediate that average prices with retention are always higher than those in the benchmark. As λ approaches zero, the price difference disappears.

Figure 4.4: Effect on average price of the possibility of making retention offers.



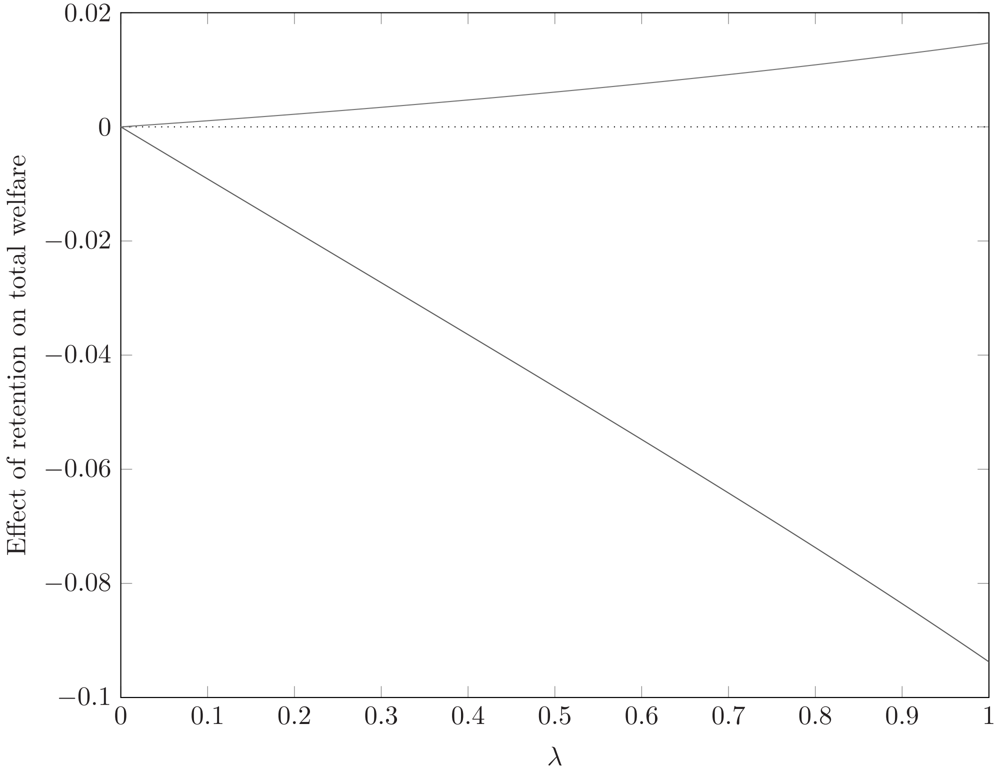
The figure gives the upper and lower bound of the effect of the possibility of retention offers on average prices paid in equilibrium, as a function of lambda.

This is intuitive: with $\lambda = 0$, the number of low types is zero, so no retention offers will be made, rendering the case with the possibility of retention offers identical to the benchmark.

The effect on total welfare

Figure 3.4 reports on an analysis that is very similar to that in Figures 3.2 and 3.3, but now for total welfare.

Figure 4.5: Effect on welfare of the possibility of making retention offers.



The figure gives the upper and lower bound of the effect of the possibility of retention offers on total welfare, as a function of lambda.

Chapter **5**

Marital Infidelity and Impatience: Infidelity and Smoking Habits*

5.1 Introduction

Infidelity is a subject of substantial importance. It is, for instance, the cause of many divorces.[1] The dissolution of a household has severe economic consequences, e.g. the loss of economics of scale, income and of future pension entitlements. Moreover, a divorce might impact household members in non-pecuniary ways as well. Children's welfare might, for instance, be negatively affected by it, see for example Weiss and Willis (1985). Even if a relationship survives a disclosed affair can have a significant impact on the utility derived from the relationship as it signals marital dissatisfaction. Additionally, it may result in re-negotiation of the marital terms. Moreover, adultery matters as it is one of the possible forms of intercourse and sex plays an important role in society. Testimonies to this are Akerlof, Yellen and Katz (1996) and Posner (1994), which address topics like contraception, abortion and shot-gun marriages. Furthermore, extramarital sex makes a substantial contribution to the spread of sexual diseases, see e.g. Pongou (2009) and Pongou and Serrano (2013).

Fair (1978) is the first economist to study infidelity. He borrows the time allocation

*This chapter is based on Siekman (2014).

[1] Cox (2008) rightfully points out that in some cases infidelity is not the cause of divorce but rather a symptom of a bad match between spouses. In those instances an individual uses it as a strategy to find a new match as the relationship will dissolve in any case. An affair will not change this outcome, but might influence its timing.

framework from Becker (1974) to develop a theory for time spent on extramarital affairs and subsequently empirical tests it. A small literature on adultery emerged from this paper. Although this literature considers a wide variety of possible determinants of infidelity, surprisingly little attention has been paid to the fact that the consequences of an affair lay in the future and thus should be discounted accordingly. A notable exception is Smith (2012). His approach is based upon the literature of economics of crime (e.g. Becker (1968)) and takes into account that cheating individuals are aware that their spouse might give a punishment once the affair is discovered, which is the rationale for them to try to conceal it. In the framework of Smith (2012) an individual's discount factor for these costs is permitted to depend on the level of education, which affects the infidelity decision through other channels as well.

This chapter extends the theoretical model of Smith (2012), by allowing a person's discount factor to vary with a person's level of impatience, and estimates it. In the empirical analysis I use smoking behavior to measure impatience. It is an empirical fact that smokers are more impatient than non-smokers, see for instance Fuchs (1982), Mitchell (1999) or Bickel, Odum and Madden (1999). Munasinghe and Sicherman (2006) and Khwaja, Sloan and Salm (2006) also establish that smoking is an indication of a high discount rate. In addition, Gränsmark (2012) shows that smokers are more impatient among chess players. There are several other studies which have used smoking habits as a measure of impatience. DellaVigna and Paserman (2005) use it in their study of impatience on job search. Other papers introduce time preferences, measured through smoking behavior, into the earnings regression to study the ability bias. (Evans and Montgomery (1994), Chevalier and Walker (2001), Fersterer and Winter-Ebmer (2003)). Pabilonia and Song (2013) find that smoking single mothers spend less time on educational activities or meals with their children than non-smokers mothers do.

I find in my empirical analysis that the estimated coefficients on the variables related to smoking habits are significant. In particular, I show that persons who (used to) smoke regularly are more likely to have an affair. This result can be explained by the fact that these people are generally more impatient. Hence, when modeling adultery the associated discounted expected costs should be taken into account. This is a suggestion Wax (2011) already made in his informal exposition on the role that time preferences play in an individual's decision on marriage and sexual behavior.

My analysis sheds, in addition, some light on the relationship between infidelity and education. Fair (1978) finds that education is negatively associated with infidelity and occupation positively. This is somewhat surprising as he uses both as a proxy for wages, and the time allocation framework cannot explain this result. In my model and that of Smith (2012) the opposing signs of education and occupation can be explained by the following mechanisms. Occupation is considered to be a measure of a person's quality as a sexual partner and the associated benefits from an extramarital affair. Occupation does not capture a person's sexual desirability fully, but still it is clearly positively correlated with the socio-economic status of an individual. Education, on the other hand, affects the likelihood of a liaison in three ways. First, schooling raises a person's social skills. Second, it affects an individual's quality as higher educated persons are able to find better jobs and higher wages, a channel Fair (1978) uses to justify schooling as a wage proxy. These two effects add to the benefits of an affair and thereby raise a person's desirability. Third, higher educated persons discount the expected costs of a liaison less. This effect works in the opposite direction, and provides the reason why infidelity might be negatively associated with schooling. Smith (2012) finds as Fair (1978) a negative relationship between schooling and adultery. I, however, show that once one controls for impatience the discounting-effect of education is (partly) filtered out. As the quality (signaling) effect of schooling becomes more dominant, the association with infidelity becomes more positive. The relationship between education and adultery even becomes positive for some estimations, thereby illustrating that it is important to control for impatience in modeling extramarital sex.

The remainder of this chapter is organized as follows: Section 5.2 presents the theoretical model. Section 5.3 gives an overview of the empirical literature related to this paper. Section 5.4 describes the data used in the study, while Section 5.5 is devoted to the methodology and consists of a discussion of the probit model, its heteroskedastic variant, and the possible causes of heteroskedasticity in this particular setting. Section 5.6 presents the estimation results and Section 5.7 concludes.

5.2 Theoretical framework

I extend the two period model suggested by Smith (2012) by introducing a variable that captures impatience. This model builds upon Liu (2008) in using an economics-of-crime-model to studying infidelity. Liu (2008) develops a dynamic model in which the possible consequences of adultery are incorporated and the discovery of an affair is stochastic. It is not unreasonable to use the economics-of-crime approach to model infidelity: in some regions an extramarital affair was actually a criminal offence for a certain period of time (see e.g. Rasmusen (2002)). Nowadays adultery is still considered undesirable by many people which justifies my approach: less than 3% of the US General Social Survey sample indicated that extramarital sex is "not wrong at all".

In the model an individual chooses a level of infidelity b to maximize the following utility function:

$$u(b, q, e, v, s) = r(b, q, e, v) - \sum_{j=1}^{J} \beta(e, s) z^j(b, v) - p(b)\beta(e, s)c(b, q). \tag{5.1}$$

Here $r(b, q, e, v)$ gives the return of infidelity and $c(b, q)$ represents the costs a person might incur if adultery is discovered. Such costs might involve the termination of the relationship with the spouse, retaliation, a change in the terms of marriage, or damage to the individual's reputation. $p(b)$ gives the likelihood that an affair is discovered. There are J other possible costs associated with an affair and which might be incurred independent of discovery. $z^j(b, v)$ captures expected costs $j \in \{1 \ldots, J\}$ and might for instance represent a sexual disease or unintended pregnancy. $\beta(e, s)$ is a discount-factor accounting for the time between the affair and its costs. q is the quality of a person and captures physical attributes, personality and economic resources. The education level of an individual is represented by e. v is a vector of variables influencing a person's opportunities for adultery (e.g. search costs), variables affecting the propensity to cheat (e.g. gender), and individual characteristics that might affect the costs of an affair (e.g. gender or religiousness). The novel aspect of this paper is that I consider the impact of a person's level of impatience, s, on the infidelity decision.

Let us consider the third term in (5.1) first. This term, the expected disutility from punishment, is the reason why a person involved in an affair will try to conceal it. The probability of getting caught $p(b)$ is assumed to be increasing in the level of adultery:

$p_b > 0$. If an extramarital relationship is discovered the offender will incur a cost $c > 0$ imposed by the spouse. The severity of such sanctions is assumed to depend on the level of adultery: $c_b > 0$. Higher quality persons have more options outside the marriage and therefore $c_q < 0$. By arguments given in Becker and Mulligan (1997) β should be allowed to depend on e, with $\beta_e > 0$ and $\beta_{ee} < 0$.[2] The intuition is that higher educated persons use higher weights for second period pay-offs as they are better able to imagine outcomes of current behavior and they are superior in mentally simulating possible scenarios. An alternative explanation, which Fuchs (1982) already provided, is that individuals with a low discount factor might invest more in education. More impatient persons discount the possible consequences of an affair more. Hence, $\beta(e, s)$ with $\beta_s < 0$ and $\beta_{ss} > 0$.[3]

The second term of (5.1) gives the J other expected costs of infidelity. $z_b^j > 0$ as a higher level of infidelity leads to higher expected costs.

The benefits of an affair (r) are increasing in the level of infidelity but at a decreasing rate: $r_b > 0$ and $r_{bb} < 0$. The quality of an individual influences the benefits of an affair as a person of higher quality is able to attract more desirable bed-partners. Hence, $r_q > 0$ and $r_{qq} < 0$. Education impacts the benefits of an affair because it improves communicative and social skills. However the model allows education to have an indirect effect through quality: $q_e > 0$. Education might, for instance, contribute positively to the economic resources of an individual.

Equation (5.1) gives rise to the following first order condition for an interior solution:

$$\frac{\partial r(b,q,e,v)}{\partial b} = \sum_{j=1}^{J} \beta(e,s)\frac{\partial z^j(b,v)}{\partial b} + p'(b)\beta(e,s)c(b,q) + p(b)\beta(e,s)\frac{\partial c(b,q)}{\partial b} \geq 0. \tag{5.2}$$

5

The marginal effect of education on the optimal level of cheating is found to be:

$$\frac{\mathrm{d}b^*}{\mathrm{d}e} = \frac{r_{be} + q_e[r_{bq} - p_b\beta(e,s)c_q - p(b)\beta(e,s)c_{bq}] - \beta_e[p_b c(b,q) + p(b)c_b + \sum_{j=1}^{J} z_b^j]}{\beta(e,s)[2p_b c_b + p(b)c_{bb} + p_{bb}c(b,q) + \sum_{j=1}^{J} z_{bb}^j] - r_{bb}}.$$

[2]In Becker and Mulligan (1997) a consumer can choose how much effort and/or resources she dedicates to imagining future outcomes. That is, a consumer faces a multi-period utility maximization problem in which $\beta(S)$ is the discount factor and she can increase S against some costs, e.g. buy a piggy bank or become a member of a Christmas Club. $\beta_{ee} < 0$ is needed to make the model compatible with the interpretation of Becker and Mulligan (1997) but it is not necessary for my analysis.

[3]$\beta_{ss} > 0$ allows my framework to be compatible with Becker and Mulligan (1997) but it is not necessary for my analysis.

(5.3)

The denominator is, as in Smith (2012), taken to be positive. For this to hold, given the discussion above, it is sufficient for the costs of infidelity (c and z) and the probability of discovery (p) to be linear in b.

There are three primary channels through which education influences the infidelity decision. First, relational skills might benefit from schooling and in turn influence b^* positively: $r_{be} > 0$. Second, education impacts quality positively ($q_e > 0$) which affects b^* positively. The term r_{bq} is positive as higher quality persons have a greater marginal return to infidelity and therefore will have more affairs. c_{bq}, on the other hand, is assumed to be negative. The reason is that additional affairs will raise the probability of detection, but the associated expected marginal costs decreases in an individual's quality. Third, educated individuals are more patient and weigh the expected costs of an affair more. The sign of the numerator of (5.3) is therefore ambiguous. However, once one controls for the impact of impatience on β this might reduce the influence of education on it. Hence, β_e might be smaller than in a model without a measure of impatience, resulting in an increase of the marginal effect of education on infidelity. In fact, this marginal effect may change from a negative to a positive one, and in the empirical part of this chapter I find some evidence for this.[4]

The effect of s on b^* is straightforwardly found to be:

$$\frac{\mathrm{d}b^*}{\mathrm{d}s} = \frac{-\beta_s[p_b c(b,q) + p(b)c_b + \sum_{j=1}^{J} z_b^j]}{\beta(e,s)[2p_b c_b + p(b)c_{bb} + p_{bb}c(b,q) + \sum_{j=1}^{J} z_{bb}^j] - r_{bb}}. \tag{5.4}$$

5

From the arguments above both numerator and denominator are positive. Therefore the model predicts that an increase in impatience leads to higher level of b^*. The intuition is that impatient persons place a lower weight on the future consequences of their actions: $\beta_s < 0$.

[4]Evidence exists that more educated persons gain more from marriage (Gustafsson and Worku (2005), Bruze, Svarer and Weiss (2012)) and have greater cross spousal productivity (Grossbard-Shechtman (1993)). The model could therefore be extended to allow for education to increase the costs of a liaison. This would add the term $-\beta(e,s)(p_B c_e + p(b)c_{be})$ to the numerator of (5.3). This makes a negative contribution to $\frac{\mathrm{d}b^*}{\mathrm{d}e}$ under the assumption that education leads to higher costs of an affair. The introduction of impatience in the discount factor would, however, generate the same qualitative effect. Another direction to extend the model is to take into account that probability of detection is decreasing in schooling as in Friehe (2008) and Lochner and Moretti (2004), thereby providing another channel through which education positively affects adultery. One could also allow education to affect z negatively. The intuition would be that the more educated are likelier to prevent the expected costs of adultery, for instance by contraception.

$\frac{\mathrm{d}b^*}{\mathrm{d}q}$ can be derived in a similar fashion and can be shown to be positive under the given assumptions. The impact of v upon the optimal level of infidelity depends on the sign of r_{bv} or z_{bv}. This results in the inference that b^* will be negatively related to search costs and factors that increase the costs of an affair, for instance strong religious beliefs.

5.3 Related literature

Besides the earlier mentioned scientific articles there are several other papers that contribute to the literature on infidelity. Cameron (2002), for instance, is closely related to my work. He includes a very wide range of variables in his estimation to explain extramarital sex. Most variables address past personal (sexual) history but he includes risk proxies as well, amongst which are fear of AIDS and whether a person is a heavy smoker and/or drinker. His estimates for the coefficients of the smoke-variables are insignificant. However, this may be caused by including dummies on a person's age when having for the first time intercourse in the estimation. In particular, the dummy variables on when a person's first sexual experience might cause this insignificance as they are a proxy for impatience. Another paper related to my work is Yamamura (2012). He finds, using Japanese data, that smokers are more likely to have a positive view on extramarital sex than non-smokers.

The other research on infidelity can be classified roughly into two groups. First, a large share of studies tried to verify the results of Fair (1978) with alternative econometric methods, (e.g. Wells (2003); Li and Racine (2004); Wang (1997); Yen (1999); Chernozhukov and Hong (2002)). Second, the other branch of literature basically suggest new factors affecting adultery. This branch includes Treas and Giesen (2000), which considers the impact of sexual tastes and values on infidelity, and Brooks and Monaco (2013), which focuses on assortive mating. Elmslie and Tebaldi (2008) studies extramarital affairs but takes a more evolutionary biological approach by focusing on the difference in adultery between men and women. Kuroki (2013) contributes by including the opportunities for infidelity as explanatory variable proxied by workplace sex ratios. Adamopoulou (2013) analyzes the seasonality and state dependence of infidelity. Potter (2011) has data on wage rates and includes these in his empirical analysis.

5.4 Data

In this study a time series of cross-sectional U.S. data from the General Social Survey (GSS) is used. The survey asks demographic, attitudinal and habitual questions. Multi-stage stratified sampling was used to obtain these data. In each stage selection was either systematic or with a probability proportional to the size of a unit. More details of the sampling procedure can be found in Smith, Marsden, Hout and Kim (2013).[5] Elmslie and Tebaldi (2008), Kuroki (2013) and Smith (2012) employ this data as well to study infidelity. My analysis is restricted to 1991, 1993 and 1994 as only in these years information on both extramarital sex and smoking habits was inquired for. The GSS consists of a set of questions asked to each subject and sets of questions asked to random subsamples. I only employ the subsamples in which questions on both smoking habit and infidelity were asked.[6]

Table 5.1 gives the selection procedure used to arrive at the dataset I work with. First, the years 1991, 1993 and 1994 were selected for reasons described above. Second, persons who never married, who are widowed or whose marital status is missing were excluded from this study. Next I selected respondents who answered the question on infidelity which reduces the sample size to 3794. The greatest part of this reduction is due to fact that the question on infidelity was not posed in each subsample. Of the 4246 respondents less than 2.5% did not answer the question on extramarital affairs, which supports the reliability of the data. The sample is further reduced by only retaining observations for which smoking data is available: there are 21 observations dropped because of incomplete response and another 2185 observations as these respondents did not receive questions on smoking habits.

Dummy variables were created to capture smoking habits. *Smoker* takes the value 1 if the respondent smokes and never attempted to quit. *AttemptQuitSmoking* equals 1 for individuals who still smoke but who have tried to give up smoking. With *ExSmoker* persons who used to smoke regularly are indicated.[7]

The infidelity dummy variable is assigned a value of 1 if a person answered affirmative

[5] http://www3.norc.org/GSS+Website/Documentation/

[6] The GSS use the terms *ballot* and *version* to indicate these subsamples. For my analysis I use ballots B and C from 1991 and 1993. From 1994 I use sample A versions 2 and 3.

[7] When respondents asked what regularly meant they were answered whatever (s)he thinks is "regularly".

Table 5.1: Sample Selection

Data	**Observations**
All data	57061
Years 1991,1993 and 1994	6115
Never married excluded	4876
Widows excluded	4248
Marital status missing	4246
Infidelity data available	3794
Smoking data available	1588

to the question: "Have you ever had sex with someone other than your husband or wife while you were married?" Of the men in the sample 20.27% admitted to have been unfaithful, of the women 13.09%. Percentages of such magnitude are not uncommon in the literature, especially for U.S. data. In other countries these figures tend to be somewhat lower. Note that most papers employing U.S. data use the GSS as well, although different years or versions. I only know of Adamopoulou (2013) who does not use these data for the U.S., he finds lower percentages. However, he uses a different definition of infidelity and only considers persons not older than 34. Adultery might be subject to measurement errors as bragging might lead to an upward bias while attempting to conceal an affair might lead to a downward bias. In addition, there might be a recall bias and respondents might not be willing to disclose this very personal and potentially embarrassing information. By employing a separate self-administered questionnaire on sensitive issues, amongst which are infidelity and sexual behavior, the survey attempted to minimize these measurement problems. As pointed out above, in the sample less than 2.5% of the respondents did not answer the question on adultery, which supports the reliability of the data.

5

Table 5.2 offers a first glance on the relationship between smoking habits and extra-marital sex without controlling for any other factors. It suggest that the probability of infidelity strongly increases with the impatience of a person, as measured by her/his smoking habit.

I am interested in the relation between education and infidelity as well. A respondent's education level is captured by categorical binary variables which indicate the highest degree a respondent attained. The reference category consists of individuals who finished high school and is indicated by the $HighSchool$ variable. The $BachelorOrMore$

Table 5.2: Smoking and infidelity

Variable	Frequency	Percentage unfaithful	
Smoker	110	28.18	
AttemptQuitSmoking	382	23.04	
ExSmoker	398	16.08	
NeverSmoked	698	10.46	
Total	1,588	16.12	$\chi^2_3 = 41.8961$ p-value < 0.0001

variable indicates a bachelor's degree or higher, *JuniorCollege* equals 1 for individuals who finished junior college, while *LessThanHighSchool* captures all individuals who did not finish high school. Table 5.3 hints at a u-shaped relation between education level and infidelity, with infidelity most common among individuals who completed high school.

Table 5.3: Education and infidelity

Variable	Frequency	Percentage unfaithful	
LessThanHighSchool	251	14.74	
HighSchool	840	18.33	
JuniorCollege	111	15.32	
BachelorOrMore	385	12.21	
Total	1,587	16.07	$\chi^2_3 = 7.8246$ p-value 0.0498

5

In the analysis I will control for several other factors that might impact the probability of infidelity. I will match these controls to those in Smith (2012) and introduce several new ones. Most variables are directly taken from the GSS, however, some variables require a more elaborate discussion.

The *Divorce* variable indicates if a person has ever been divorced or legally separated. In the dataset 6 persons indicated that they divorced or separated at least once, while their answers to other questions contradict this. For these persons *Divorce* is recorded as missing.

To allow all persons in his sample to have completed their education and have chosen their job Smith (2012) does not consider individuals aged under 25 in his estimation. I do not exclude these persons in my analysis here, however, I performed robustness checks without them as well, leading to no significant difference in findings. I allow age to have a non-linear impact on the probability of an affair. In addition, the year of birth is included in the analysis.

In my analysis I control for occupation and associated status by making use of International Socio Economic Index (ISEI) developed by Ganzeboom, De Graaf, and Treiman (1992). This is a continuous measure of socio-economic status and reflects the income and educational background of an occupation. The ISEI score can be matched to an individual's 1988 ISCO-code (International Standard Classification of Occupations), which is developed by ILO and which is provided in the dataset. I use the latest version of ISEI (ISEI-08: Ganzeboom and Treiman (2011)), which estimates the index using both male and female respondents. Earlier versions only used data on males to calculate ISEI scores. The new index should therefore be a better reflection of a person's socio-economic status, especially in female dominated occupations. Note that ISEI differs from occupational prestige scales, as the latter are based upon subjective judgements of "the general desirability of occupations", Goldthorpe and Hope (1972), Goldthorpe and Hope (1974).

Four dummy variables were created to control for an individual's employment status. $FulEmp$ equals 1 for those who work fulltime and is the reference category. $PartEmp$ indicates that a person works part-time or (s)he has a job, but is not at work because of, for instance, temporary illness, vacation, or strike. $UnEmp$ groups all respondents who are unemployed or retired. $OtherEmpStatus$ captures all other persons, for instance, those who are at school or keeping house.

I measure how religious an individual is on a four point scale. A person scores 1 if (s)he never attends a religious service. A score of 2 is obtained if attending once per year or less, while attending more than this but less than once per week results in a 3. A 4 is given to persons who attend services once per week or more. $LogPopulation$ is the log of the population size (rounded to the nearest thousand) of the place where the respondent lives. Of places where the population was rounded to zero the log of 0.25 was taken. It is incorporated in my estimations as well as a proxy for the opportunity of infidelity. The $Children$ variable gives the number of children present in the household.

One potential issue might be that (ex-)smokers are more outgoing and sociable than non-smokers and therefore have more opportunities for infidelity. As a form of sensitivity analysis I therefore will include the $SocBar$ variable in my specification. This variable measures how often a person goes to a tavern or bar: never (assigned value 1), once per year or less (assigned 2), more than this but less than once per week (assigned 3) or once per week or more (assigned value 4). Unfortunately, data on this variable is only available for a small subsample, so it is not included in the main analysis.

Except the variables discussed, all other controls are dummy variables. Table 5.4 provides some descriptive statistics on the variables involved in the analysis. The differences between the conditional and unconditional means indicates the importance of controlling for these variables. Illustrative is, for instance, that the percentage females is much higher under the faithful people. Moreover, the frequency of bar and tavern visits is higher amongst those who admitted to have been unfaithful.

Table 5.4: Descriptive statistics

Variable	Unconditional			Faithful		Unfaithful	
	Obs	**Mean**	**SD**	**Mean**	**SD**	**Mean**	**SD**
Infidelity	1588	0.161	0.368				
Female	1588	0.577	0.494	0.598	0.490	0.469	0.500
Age	1585	45.232	14.557	45.498	15.001	43.852	11.927
Black	1588	0.092	0.289	0.080	0.272	0.152	0.360
OtherRace	1588	0.038	0.191	0.038	0.192	0.035	0.185
SocioEconomicIndex	1545	46.002	20.398	46.574	20.424	43.055	20.042
LessThanHighschool	1587	0.158	0.365	0.161	0.367	0.145	0.353
HighSchool	1587	0.529	0.499	0.515	0.500	0.604	0.490
JuniorCollege	1587	0.070	0.255	0.071	0.256	0.067	0.250
BachelorOrMore	1587	0.243	0.429	0.254	0.435	0.184	0.389
Religion	1559	2.766	1.025	2.822	1.0165	2.468	1.022
LogPopulation	1588	3.220	2.224	3.184	2.256	3.410	2.044
Children	1578	0.902	1.149	0.915	1.148	0.835	1.154
Divorce	1582	0.393	0.488	0.340	0.474	0.665	0.473
FulEmp	1588	0.5516	0.4975	0.5435	0.4983	0.5938	0.4921
PartEmp	1588	0.1215	0.3269	0.1201	0.3252	0.1289	0.3358
UnEmp	1588	0.1341	0.3409	0.1336	0.3404	0.1367	0.3442
OtherEmpStatus	1588	0.1927	0.3945	0.2027	0.4022	0.1406	0.3483
SocBar	817	1.8849	1.0032	1.7980	0.9764	2.3488	1.0205

Obs gives the number of observations
SD gives the Standard Deviation

5.5 Methodology

To determine the relationship between infidelity and the in Section 5.4 suggested variables I estimate a probit model. Assuming that the same theoretical factors are involved in the determination of the likelihood and quantity of infidelity, the model above suggest to include individual quality and education amongst the set of control variables. Several model-specifications will be estimated with a varying set of controls because some of these might have an endogeneity issue (e.g. whether a person has children).

I extend the analysis by considering a heteroskedastic probit model. Let y denote the dichotomous variable indicating whether an individual is involved in an affair. In the homoskedastic probit model

$$y = \begin{cases} 1 \text{ if } \boldsymbol{x}'\boldsymbol{\beta} + u > 0 \\ 0 \text{ if } \boldsymbol{x}'\boldsymbol{\beta} + u \leq 0, \end{cases}$$

where $\boldsymbol{x}$ is a vector of explanatory variables and the error term u is standard normal distributed. Therefore $\Pr[y = 1|\boldsymbol{x}] = \Phi(\boldsymbol{x}'\boldsymbol{\beta})$. In the heteroskedastic model that I consider $u \sim N(0, \exp(2\boldsymbol{z}'\boldsymbol{\gamma}))$ and consequently

$$\Pr[y = 1|\boldsymbol{x}, \boldsymbol{z}] = \Phi\left(\frac{\boldsymbol{x}'\boldsymbol{\beta}}{\exp(\boldsymbol{z}'\boldsymbol{\gamma})}\right).$$

One of the factors used to model heteroskedasticity is gender because there is a difference in infidelity strategies between men and women according to the literature, see e.g. Cox (2008) and Elmslie and Tebaldi (2008). There it is argued that women are more reluctant to commit adultery as they put a higher valuation on the resources provided by the partner to the relationship (and children). Females are only willing to put the relationship at risk when the potential bed-partner has good genes or is wealthy. The probability that males cheat, on the other hand, is less dependent on these factors. As they do not value a stable relationship as high they are on average less faithful. All together this means that females might display higher variation in (expected dis-)utility derived from infidelity and its possible consequences. The age of an individual is part of the variance equation as well. The reason is that young people are more attractive as a possible candidate for a liaison, but as age progresses not only the level but also the variation in attractiveness drops. An indicator variable whether a person has ever been divorced or legally separated is included in $\boldsymbol{z}$ too. Divorced persons might be likelier to have an affair as they might have had an affair before and that was the reason they got divorced in the first place. On the other hand, divorce might have been a traumatic experience for an individual and (s)he might want to avoid going through it again, thus reducing the likelihood of adultery. Hence, divorced persons exhibit a greater variation in their evaluation of infidelity.

5.6 Estimation results

This section first presents the results from the main analysis. Subsequently, estimations only using individuals in their first marriage are shown, followed by estimations controlling for the number of bar/tavern visits. These last two estimations are not included in the main analysis as they reduce the number of observations substantially.

All estimations are weighted by sampling probability weights. Standard errors are corrected for the survey-design as suggested in the documentation provided with the GSS dataset. These standard errors take stratification and the clustered nature of the sample into account. Variables for stratification and clustering, as well as the sampling weights, were directly taken from the GSS dataset. Strata containing a single sampling unit were assigned to a neighboring stratum.

In Table 5.5 the results of several probit estimations are presented along with the marginal effects. In the first column I only include basic individual characteristics besides the variables that capture smoking habits. Specification (2) is a benchmark with exactly the same variables as in Smith (2012). The only differences between my analysis and his is the sample, as I want only individuals on which there is data on smoking habits, and that he does separate estimations for men and women. I refrain from this separation as the number of observations is rather small. The third model extends this benchmark with the smoke variables. The fourth estimation drops the number of children from the specification, as it might suffer from an endogeneity issue. The fifth specification includes it along with an indicator of ever being divorced. In (6) I include variables on occupation status. Column 7 estimates a heteroskedastic probit model as a robustness check.

A subset of smoking-habit variables is statistically significant in all presented specifications. Remarkably, individuals who smoke and never attempted to quit have a higher likelihood of having an affair than those who made such an attempt. The coefficients of both these groups are larger than that for ex-smokers, who in turn have a higher coefficient than individuals who never smoked. These results provide evidence for the hypothesis that impatient persons are likelier to have an affair.[8] It is interesting to

[8]An alternative explanation is that smokers don't discount the costs of an affair less, but underestimate these costs all the time. However, this seems less plausible.

Table 5.5: Probit estimates of marital infidelity

Dependent variable: extramarital affair	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Smoking habits. Reference category: Never smoked							
Smoker	0.5922***		0.4772***	0.4956***	0.4579***	0.4510***	0.5816***
	(0.1489)		(0.1587)	(0.1595)	(0.1597)	(0.1602)	(0.2190)
M.E.	0.1616		0.1226	0.1285	0.1108	0.1088	0.1179
AttemptQuitSmoking	0.4669***		0.3895***	0.3787***	0.3061***	0.2981**	0.4496**
	(0.1040)		(0.1106)	(0.1094)	(0.1143)	(0.1163)	(0.1992)
M.E.	0.1141		0.0916	0.0891	0.0671	0.0651	0.0831
ExSmoker	0.2161**		0.1736	0.1722	0.1954*	0.1981*	0.2051
	(0.0972)		(0.1054)	(0.1054)	(0.1114)	(0.1108)	(0.1482)
M.E.	0.0495		0.0385	0.0383	0.0414	0.0420	0.0337
Female	-0.2490***	-0.2937***	-0.2537***	-0.2515***	-0.2964***	-0.3150***	-2.5777
	(0.0814)	(0.0797)	(0.0811)	(0.0819)	(0.0864)	(0.0910)	(3.0881)
M.E.	-0.0553	-0.0648	-0.0550	-0.0547	-0.0613	-0.0652	-0.0761
SocioEconomicIndex		-0.0013	-0.0005	-0.0001	0.0004	0.0008	0.0021
		(0.0026)	(0.0026)	(0.0026)	(0.0027)	(0.0027)	(0.0029)
M.E.		-0.0003	-0.0001	-0.0000	0.0001	0.0002	0.0003
Religion		-0.1811***	-0.1466***	-0.1539***	-0.1263***	-0.1248***	-0.0989**
		(0.0382)	(0.0385)	(0.0387)	(0.0399)	(0.0392)	(0.0486)
M.E.		-0.0392	-0.0313	-0.0330	-0.0257	-0.0254	-0.0159
LogPopulation		0.0188	0.0191	0.0204	0.0121	0.0127	0.0199
		(0.0212)	(0.0216)	(0.0213)	(0.0215)	(0.0219)	(0.0213)
M.E.		0.0041	0.0041	0.0044	0.0025	0.0026	0.0032
Education. Reference category: High School							
LessThanHighschool		-0.2340*	-0.2854**	-0.2627**	-0.2704**	-0.2752**	-0.3044*
		(0.1197)	(0.1201)	(0.1174)	(0.1263)	(0.1271)	(0.1602)
M.E.		-0.0465	-0.0549	-0.0512	-0.0500	-0.0508	-0.0437
JuniorCollege		-0.1177	-0.1097	-0.1050	-0.1406	-0.1350	-0.1780
		(0.1827)	(0.1842)	(0.1849)	(0.1907)	(0.1875)	(0.2285)
M.E.		-0.0242	-0.0224	-0.0215	-0.0270	-0.0260	-0.0262
BachelorOrMore		-0.2239*	-0.1832	-0.1982	-0.1320	-0.1294	-0.2254
		(0.1204)	(0.1223)	(0.1243)	(0.1279)	(0.1278)	(0.1499)
M.E.		-0.0459	-0.0374	-0.0405	-0.0263	-0.0255	-0.0346
Children		-0.0650*	-0.0655*		-0.0352	-0.0391	-0.0375
		(0.0367)	(0.0366)		(0.0383)	(0.0386)	(0.0480)
M.E.		-0.0141	-0.0140		-0.0072	-0.0080	-0.0060
Divorce					0.6144***	0.6180***	0.0889
					(0.0949)	(0.0970)	(0.1545)
M.E.					0.1384	0.1391	0.1364
Employment Status. Reference category: Full time employed							
PartEmp						0.1136	-0.1284
						(0.1314)	(0.1676)
M.E.						0.0240	-0.0194
UnEmp						0.1429	0.0088
						(0.1719)	(0.1821)
M.E.						0.0305	0.0014
OtherEmpStatus						0.0855	0.2564
						(0.1412)	(0.3316)
M.E.						0.0178	0.0463
Variance equation							
Female							1.2231
							(0.8519)
Age							-0.0096*
							(0.0052)
Divorce							0.5730***
							(0.1433)
No. of observations	1585	1503	1503	1513	1497	1497	1497
Wald test joint significance							
Smoking habits	8.8977***		5.1064***	5.1359***	3.9286***	3.7268**	2.4853*
Education		2.2478*	2.4883 *	2.3550*	1.860	1.8682	1.5268

All estimations include a constant term, age, age squared, year of birth and controls for ethnicity. Estimates for these coefficients are not reported here but are available upon request. Standard errors in parentheses. M.E. is shorthand for Marginal Effect, averaged over individuals. Note that for binary variables these effects are the difference in the predicted probability of an affair when the variable changes from 0 to 1.
* significant at 10% level; ** significant at 5% level; *** significant at 1% level.

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compare the marginal effects of the first specification with the differences in percentages in Table 5.2 between a certain group and the persons who never smoked. The table indicates, for instance, that amongst smokers who never made a quit-attempt 17.72% $(28.18 - 10.46)$ more persons had an affair. The estimated marginal effect for this group is of the same magnitude, 16.16%. Similar findings hold for the other groups, although the differences between the marginal effects and the percentages that can be derived from Table 5.2 become somewhat larger once one includes more control variables.

Comparing specifications (2) and (3) provides some evidence that controlling for impatience affects the relation between education and infidelity. In particular, once impatience is incorporated, the coefficient on *Bachelor* reduces in magnitude and becomes insignificant. On the other hand, the coefficient on $LessThanHighschool$ amplified. The marginal effects display similar effects. The reasoning behind these changes is that, once one controls for impatience, the role the educational level plays in explaining the discount factor is reduced. Hence the ability of education to signal quality and enhanced relational skills now dominates the impact education has on the likelihood of having an extramarital affair. Overall, this results in a more positive relation between education and the probability of infidelity than as suggested by benchmark specification (2).

From Table 5.5 one can conclude that men are more likely to be involved in an extramarital affair than women, which is in line with existing literature. Reasons might be that women bear the risk of pregnancy, might have greater moral or religious constraints, and might have a more difficult time on the remarriage market. Moreover, I find that more religious persons are less likely to have a paramour. The variables on employment status are insignificant, which might be attributed to the fact that the model already included a socio-economic index.

There is some evidence that the number of children is negatively associated with the probability of infidelity. *Divorce* has a highly significant positive coefficient in the fifth specification. However, both these variables might be endogenous. An initial bad match between spouses might reduce the likelihood of having children and increase the probability of both an affair and divorce. Consequently, no causal interpretation can be given to these results. I include these variables, however, to make comparison possible to the results found in the literature.

To verify the validity of the model I performed a normality test on specification (6) and found that normality could not be rejected. As a robustness check I extended the analysis with specification (7) which allows for heteroskedasticity. A Wald-test suggests joint significance of the variance equation. Clearly *Divorce* seems to be the driving force behind heteroskedasticity. Moreover, younger persons seem to display somewhat more hetereoskedasticity. There are not many differences between the estimated level-coefficients of homoskedastic and heteroskedastic specifications (6) and (7). First, the estimated female and divorce coefficients are no longer significant, but this might be due to their inclusion in both the level and variance equation. Second, the estimated coefficients on the employment status change somewhat, but they remain insignificant. Most importantly, when controlling for heteroskedasticity the relation between infidelity and smoking remains significant. However, the difference between current smokers and other individuals becomes larger.

Including divorced persons in the analysis might cause both endogeneity and heterogeneity issues. I therefore re-do the analysis for persons who are in their first marriage. Table 5.6 presents the results of this exercise. The subsample of individuals in their first marriage is rather small and the percentage unfaithful has dropped from 16.1% (see Table 5.4) to 8.9%, both affecting the precision of the estimates negatively. No heteroskedastic specification is reported, although it was estimated, but the variance equation became insignificant as *Divorce* was no longer part of it.

Again I find that impatience, as measured by smoking habits, is significant in explaining infidelity. As before females and religious persons are less likely to have an affair. Once one controls for impatience, the effect of education on infidelity is altered in the same directions as before and the relationship between these variables becomes significant. However, the effects are rather small for this subsample. As a final robustness check I include a variable on the frequency of bar/tavern visits in my analysis. As not all respondents were interviewed on this topic, this reduces the sample size significantly. Table 5.7 gives the estimations. I present both results on the entire sample and on the sample of persons in first marriage. Moreover, each specification is estimated with and without *SocBar* to first see what the effect is of using a subsample, rather than the full one, and to see the effect of the variable's introduction. The estimations are the equivalents of specifications (1) and (6) of Table 5.5.

Table 5.6: Probit estimates of marital infidelity for persons in 1st marriage

Dependent variable: extramarital affair	(1)	(2)	(3)	(4)	(5)
Smoking habits. Reference category: Never smoked					
Smoker	0.8642***		0.8589***	0.8585***	0.8633***
	(0.2091)		(0.2165)	(0.2155)	(0.2193)
M.E.	0.1964		0.1904	0.1899	0.1250
AttemptQuitSmoking	0.4099***		0.3938**	0.3921**	0.3988**
	(0.1569)		(0.1620)	(0.1628)	(0.1658)
M.E.	0.07042		0.0663	0.0658	0.0577
ExSmoker	0.2795*		0.2651*	0.2528	0.2689*
	(0.1530)		(0.1584)	(0.1606)	(0.1554)
M.E.	0.0443		0.0414	0.0392	0.0390
Female	-0.2725**	-0.3444***	-0.2965**	-0.2983**	-0.2859**
	(0.1289)	(0.1183)	(0.1271)	(0.1282)	(0.1356)
M.E.	-0.0405	-0.0525	-0.0436	-0.0437	-0.0414
SocioEconomicIndex		0.0040	0.0048	0.0046	0.0055
		(0.0042)	(0.0042)	(0.0043)	(0.0043)
M.E.		0.0006	0.0007	0.0007	0.0008
Religion		-0.1771***	-0.1347**	-0.1335**	-0.1247**
		(0.05674)	(0.0575)	(0.0573)	(0.0567)
M.E.		-0.0266	-0.0196	-0.0194	-0.0181
LogPopulation		0.0092	0.0091	0.0095	-0.0043
		(0.0270)	(0.0285)	(0.0285)	(0.0289)
M.E.		0.0014	0.0013	0.0014	0.0006
Education. Reference category: High School					
LessThanHighschool		-0.2192	-0.2985	-0.3058	-0.3059
		(0.2008)	(0.2017)	(0.2035)	(0.2029)
M.E.		-0.0298	-0.0381	-0.0387	-0.0443
JuniorCollege		-0.2547	-0.2455	-0.2476	-0.2324
		(0.2797)	(0.2940)	(0.2946)	(0.2995)
M.E.		-0.0332	-0.0313	-0.0314	-0.0336
BachelorOrMore		-0.4556**	-0.4302**	-0.4315**	-0.4271**
		(0.2097)	(0.2116)	(0.2118)	(0.2109)
M.E.		-0.0609	-0.0561	-0.0561	-0.0618
Children		0.0221	0.0326		0.0238
		(0.0582)	(0.0567)		(0.0589)
M.E.		0.0033	0.0048		0.0034
Employment Status. Reference category: Full time employed					
PartEmp					-0.1150
					(0.1954)
M.E.					-0.0166
UnEmp					0.3514
					(0.2493)
M.E.					0.0509
OtherEmpStatus					0.0981
					(0.1821)
M.E.					0.01424
No. of observations	961	917	917	923	917
Wald test joint significance					
Smoking habits	6.4591***		5.7841***	5.7512***	5.6508***
Education		2.0158	2.1358*	2.1701*	2.1440*

All estimations include a constant term, age, age squared, year of birth and controls for ethnicity. Estimates for these coefficients are not reported here but are available upon request. Standard errors in parentheses. M.E. is shorthand for Marginal Effect, averaged over individuals. Note that for binary variables these effects are the difference in the predicted probability of an affair when the variable changes from 0 to 1.
* significant at 10% level; ** significant at 5% level; *** significant at 1% level.

Table 5.7: Probit estimates of marital infidelity when controlling for social behavior

Dependent variable: extramarital affair	**Entire sample** (1)	(2)	(3)	(4)	**Respondent in 1st marriage** (5)	(6)	(7)	(8)
Smoking habits. Reference category: Never smoked								
Smoker	0.5592***	0.5489**	0.5146**	0.5043**	0.9290***	0.9203***	0.9123***	0.8957***
	(0.2152)	(0.2208)	(0.2190)	(0.2245)	(0.2650)	(0.2676)	(0.2698)	(0.2730)
M.E.	0.1485	0.1399	0.1260	0.1201	0.2242	0.2174	0.1401	0.1355
AttemptQuitSmoking	0.4231***	0.3583**	0.3186**	0.2544	0.5042**	0.4687**	0.4971**	0.4521**
	(01488)	(0.1575)	(0.1551)	(0.1605)	(0.2086)	(0.2112)	(0.2267)	(0.2292)
M.E.	0.1003	0.0808	0.0695	0.0534	0.0943	0.0851	0.0764	0.0684
ExSmoker	0.1428	0.1118	0.0688	0.0484	0.1122	0.0786	0.0171	-0.0155
	(0.1562)	(0.1594)	(0.1616)	(0.1605)	(0.2545)	(0.2578)	(0.2342)	(0.2318)
M.E.	0.0315	0.0237	0.0141	0.0096	0.0181	0.0124	0.0026	-0.0023
Female	-0.2525**	-0.1489	-0.1918	-0.1364	-0.1822	-0.1340	-0.1382	-0.1027
	(0.1107)	(0.1170)	(0.1336)	(0.1390)	(0.1632)	(0.1703)	(0.1753)	(0.1821)
M.E.	-0.0549	-0.0311	-0.0392	-0.0271	-0.0289	-0.0209	-0.0212	-0.0155
SocioEconomicIndex			0.0037	0.0047			0.0051	0.0057
			(0.0038)	(0.0038)			(0.0053)	(0.0053)
M.E.			0.0008	0.0009			0.0008	0.0009
Religion			-0.1731***	-0.1525**			-0.1104	-0.0976
			(0.0635)	(0.0649)			(0.0789)	(0.0815)
M.E.			-0.0349	-0.0301			-0.0170	-0.0148
LogPopulation			0.0120	0.0075			0.0113	0.0076
			(0.0255)	(0.0255)			(0.0320)	(0.0314)
M.E.			0.0024	0.0015			0.0017	0.0011
Education. Reference category: High School								
LessThanHighschool			-0.1790	-0.1298			-0.4015	-0.3360
			(0.1802)	(0.1814)			(0.2589)	(0.2521)
M.E.			-0.0339	-0.0245			-0.0617	-0.0508
JuniorCollege			-0.2995	-0.3115			-0.4773	-0.4681
			(0.2415)	(0.2349)			(0 .3594)	(0.3574)
M.E.			-0.0530	-0.0538			-0.0733	-0.0708
BachelorOrMore			-0.0877	-0.1423			-0.3594	-0.3909
			(0.1671)	(0.1694)			(0.2471)	(0.2489)
M.E.			-0.0173	-0.0272			-0.0552	-0.0592
Children			-0.0289	-0.0078			0.0037	0.0158
			(0.0553)	(0.0554)			(0.0880)	(0.0878)
M.E.			-0.0058	-0.0015			0.0006	0.0024
Divorce			0.5071***	0.4810***				
			(0.1175)	(0.1169)				
M.E.			0.1114	0.1028				
Employment Status. Reference category: Full time employed								
PartEmp			-0.0689	-0.0479			-0.1455	-0.1335
			(0.1674)	(0.1749)			(0.2659)	(0.2732)
M.E.			-0.0136	-0.0093			-0.0224	-0.0202
UnEmp			0.1658	0.2249			0.5067*	0.5471**
			(0.2297)	(0.2264)			(0.2756)	(0.2745)
M.E.			0.0355	0.0480			0.0778	0.0828
OtherEmpStatus			-0.0967	-0.0189			0.0489	0.0911
			(0.2041)	(0.2094)			(0.2840)	(0.2850)
M.E.			-0.0189	-0.0037			0.0075	0.0138
SocBar		0.2660***		0.2231***		0.1747**		0.1737*
		(0.0556)		(0.0587)		(0.0846)		(0.0913)
M.E.		0.0549		0.0440		0.0270		0.0263
No. of observations	815	815	782	782	494	494	481	481
Wald test joint significance								
Smoking habits	3.7776**	2.8983**	2.6203*	2.0653	5.0206***	4.7156***	4.6062***	4.2371***
Education			0.7183	0.7452			1.7670	1.6601

All estimations include a constant term, age, age squared, year of birth and controls for ethnicity. Estimates for these coefficients are not reported here but are available upon request. Standard errors in parentheses. M.E. is shorthand for Marginal Effect, averaged over individuals. Note that for binary variables these effects are the difference in the predicted probability of an affair when the variable changes from 0 to 1.

* significant at 10% level; ** significant at 5% level; *** significant at 1% level.

The relationship between impatience, proxied by smoking, and infidelity is again significant. When *SocBar* is included the size of the smoke-variables is somewhat reduced, which might be explained by the fact that they might capture how social a person is as well, but they remain substantial and significant. Remarkably, some variables, e.g. *Female* and *Religion*, are now insignificant for some specifications, which might be caused by the limited sample size.

5.7 Conclusions

In this chapter I estimated several models for infidelity. I found that (ex)smokers have a significant higher probability of having an affair. Results suggest that smokers who never made an attempt to quit are the most likely to have an extramarital affair, the likelihood for smokers who did make such an attempt is somewhat smaller, followed by ex-smokers and persons who never smoked, respectively. The link between smoking habits and impatience is a well-established empirical fact in the literature. Consequently, my findings imply that more impatient persons are likelier to be involved in affairs. One might argue that smokers are more sociable and this drives my findings. However, when controlling for the sociability of individuals, as measured by the number of bar and tavern visits, the results on smoking survive.

The second contribution of this paper revolves around the relationship between education and adultery. Other studies have found that the likelihood of a liaison decreases with the level of education and Smith (2012) argues that this is caused by more educated persons having a higher discount factor. I introduce impatience in the theoretical framework and show that this (partly) filters out the discounting effect of education. This means that the quality signaling role of education, caused by more schooled persons having on average more economic resources, becomes more dominant. The effect that education might attribute to a person's relational skills becomes more important as well. Overall the introduction of impatience leads to a more positive effect of education on the likelihood of infidelity. The empirical findings support these theoretical results. In some estimations the effect of education on the probability of adultery actually is positive rather than negative.

All in all, I showed that expected costs of a liaison and impatience play a significant role and should be incorporated when modeling infidelity.

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Samenvatting (Summary in Dutch)

Economie is een wetenschap die de verdeling van schaarse goederen bestudeert. Er zijn veel economische modellen ontwikkeld om te onderzoeken hoe men deze goederen efficiënter kan verdelen. Pas sinds de tweede helft van de vorige eeuw is echter het besef gekomen dat er belangrijke fricties actief zijn op markten die leiden tot marktonvolkomenheden, falen of een substantiële herverdeling van welvaart. Dit proefschrift richt haar focus op twee van zulke fricties, namelijk zoek- en overstapkosten, en hun interactie met andere economische fenomenen.

Zoekkosten zijn kosten die voor rekening van de consument komen om informatie over een product te verkrijgen en de prijs ervan te achterhalen. Deze kosten kunnen verschillende vormen aannemen. Voorbeelden zijn: benzinekosten die gemaakt worden om een winkel te bezoeken, tijd die besteed wordt aan het lezen van een productomschrijving of de energie- en internetkosten voor het bezoeken van websites. Overstapkosten, daarentegen, zijn kosten die men maakt om van leverancier te wisselen wanneer men al op de hoogte is van de productspecificaties en de prijs van het product. Deze kosten kunnen bestaan uit een boete vanwege het vroegtijdig beëindigen van een contract of de administratieve kosten en moeite om partijen te informeren over een nieuw telefoon- of bankrekeningnummer. Een ander voorbeeld van overstapkosten is aanwezig in de trouwmarkt, waar het wisselen van partner kan leiden tot verlies van inkomen of het contact met de kinderen.

Dit proefschrift heeft tot doel inzicht te verschaffen in de processen die spelen op markten waar zoek- en overstapkosten een significante rol spelen. Het proefschrift is opgesplitst in bestaat uit twee delen. Het eerste deel bevat twee hoofdstukken waarin onderzoek wordt gedaan naar het zoekgedrag van consumenten. In het tweede deel worden twee hoofdstukken gepresenteerd die zich richten op markten met overstapkosten.

Hoofdstuk 2 van dit proefschrift bestudeert de interactie tussen zoekkosten en *loss aversion*. Loss aversion is het concept dat een consument additionele (dis)utiliteit ervaart wanneer een uitkomst negatief (positief) afwijkt van een referentiepunt, hetgeen vaak is gebaseerd op de verwachtingen van een consument.

Een deel van de bestaande literatuur onderzoekt al hoe bedrijven deze aversion uitbuiten. Deze veronderstelt echter dat consumenten op de hoogte zijn van alle aanbiedingen op de markt. Consumenten moeten dus gelijktijdig alle producten die worden aangeboden evalueren, en dus meteen bepalen of zij deze en de voorgestelde prijzen mee of tegen vinden vallen. Een setting waarin consumenten sequentieel de producten en bijbehorende prijzen beoordelen zou enigszins natuurlijker zijn. Een consument kan dan teleurgesteld zijn als het product dat ze overweegt of de bijbehorende prijs slechter is dan gedacht, en gebaseerd daarop kan ze beslissen of ze nog verder wil zoeken naar een betere aanbieding.

In hoofdstuk 2 wordt een dergelijke setting bestudeerd. In het daar gepresenteerde model zoeken consumenten sequentieel tegen bepaalde kosten op een markt met gedifferentieërde producten en kan een consument teleurgesteld (of blij verrast) zijn als ze op een product of prijs stuit die tegenvalt (of meevalt).

Een van de bevindingen in hoofdstuk 2 is dat loss aversion leidt tot een scala aan equilibrium prijzen. De bovengrens van dit scala stijgt in de mate van loss aversion, maar in tegenstelling tot de bestaande literatuur kan de ondergrens hier in dalen. Dus, loss aversion kan leiden tot lagere prijzen.

Loss aversion kan via drie dimensies consumenten beïnvloeden. Ten eerste in de prijsdimensie: consumenten zijn teleurgesteld wanneer ze op een hogere prijs stuiten dan verwacht. Afzonderlijk beschouwd leidt deze dimensie tot lagere prijzen omdat bedrijven meer terughoudend zijn met prijsverhogingen. Ten tweede heeft loss aversion invloed via de zoek-dimensie. Consumenten kunnen teleurgesteld zijn als ze meer moeten zoeken dan verwacht. Afzonderlijk beschouwd leidt loss aversion in deze dimensie tot hogere prijzen omdat het effectief de zoekkosten verhoogt. Ten derde heeft loss aversion een effect in de dimensie die meet hoe goed een product bij een consument past, de match-dimensie. Loss aversion in deze dimensie leidt tot een grotere set aan overeenstemmingswaarden en dus tot meer productdifferentiatie. Dit leidt tot hogere prijzen.

Al met al heeft loss aversion een ambigu effect op prijzen. Daarom wordt ook een equilibrium verfijning geïntroduceerd in hoofdstuk 2. De unieke prijs die deze verfijning overleeft daalt in de mate van loss aversion voor niet al te lage waardes. Een andere bevinding in hoofdstuk 2 is dat consumenten met loss aversion meer zoeken wanneer zoekkosten relatief laag zijn, maar minder als deze hoog zijn.

Hoofdstuk 3 draagt bij aan de literatuur over het zoekgedrag van consumenten op markten met gedifferentieërde producten. De literatuur neemt aan dat consumenten op een dergelijke markt willekeurig zoeken terwijl in hoofdstuk 3 de zoekvolgorde afhangt van de voorkeuren van de consument. Ter illustratie, een consument die op zoek is naar een woning in de nabijheid van haar werk zal niet aan de andere kant van de stad beginnen met zoeken. Vaak helpt een platform consumenten geschikte producten te vinden. Een consument vult haar voorkeuren bijvoorbeeld in op een zoekmachine of vergelijkingssite en het platform suggereert dan een verkoper die het meest waarschijnlijk het beste aan de wensen van de consument kan voldoen. Als een consument dan een product, in dit voorbeeld een huis, vervolgens heeft bekeken, kan zij beslissen om het te kopen of nog even verder te zoeken, bijvoorbeeld omdat de indeling van het huis niet naar haar smaak is. In het model van hoofdstuk 3 beginnen consumenten dus te zoeken daar waar zij het best geschikte product verwachten, gebaseerd op het advies van een derde partij.

Het hoofdstuk slaat een brug tussen twee extreme gevallen in de consumenten-zoekliteratuur. Aan de ene kant omvat het model van hoofdstuk 3 het model waar consumenten geheel willekeurig zoeken, maar het omvat ook de situatie waar consumenten perfect geïnformeerd zijn over alle producten op de markt voor bepaalde waardes van de parameters.

Een belangrijke bevinding van het model van hoofdstuk 3 is dat prijzen en winsten hoger zijn in vergelijking met modellen waarin consumenten geheel willekeurig zoeken. De reden is dat het onwaarschijnlijker is dat consumenten een concurrent bezoeken als ze eerst de verkoper bezoeken waar ze het product verwachten te vinden dat het beste aansluit bij hun voorkeuren. Dit maakt dat ex-ante producten minder homogeen zijn voor consumenten. Daarnaast laat het hoofdstuk zien dat prijzen stijgen in zoekkosten, wat in lijn is met de bestaande literatuur.

Hoofdstuk 3 presenteert ook een aantal bevindingen aangaande welvaart. Consumenten verkrijgen gemiddeld genomen een product dat beter bij hun voorkeuren past wanneer ze bij het bepalen van de zoekvolgorde rekening houden met deze voorkeuren. Bovendien stoppen consumenten, in elke fase van het proces, eerder met zoeken. Dit komt omdat zij zich realiseren dat ze eerst het product bekijken dat in bepaalde gebieden het beste bij hun voorkeuren past. Hieruit volgt ook meteen dat consumenten minder uitgeven aan zoekkosten wanneer zij de zoekvolgorde laten afhangen van hun voorkeuren. Deze twee effecten tezamen leiden ertoe dat de totale welvaart hoger is onder het geïntroduceerde zoekproces. Consumenten zijn echter slechter af dan in de situatie wanneer ze willekeurig zouden zoeken omdat deze effecten teniet worden gedaan door de hogere prijzen die producenten kunnen vragen.

De resultaten in hoofdstuk 3 staan haaks op enkelen in de literatuur die zijn afgeleid aan de hand van een model waarin een verkoper prominent is, en in tegenstelling tot het model in hoofdstuk 3, door alle consumenten als eerste bezocht wordt. In een dergelijk model is de prijs van de eerste verkoper lager dan dat van de concurrenten en lager dan in het geval van willekeurig zoekgedrag omdat de vraag elastischer is. In hoofdstuk 4 zet deze verkoper dezelfde prijs als haar concurrenten welke hoger ligt dan in het geval van willekeurig zoeken. Daarnaast verkrijgen consumenten in een dergelijk model gemiddeld genomen een product dat verder van hun voorkeuren afligt omdat ze vaker genoegen nemen met minder vanwege de lagere prijs van de prominente verkoper. In een dergelijk raamwerk is de totale welvaart lager dan in het geval van willekeurig zoekgedrag, terwijl in hoofdstuk 4 de introductie van zoeken gebaseerd op voorkeuren leidt tot hogere welvaart. Dit leidt tot een belangrijke beleidsimplicaties. Beschouw een scenario waarin een platform de totale welvaart op een markt naar zich toe kan trekken door kosten in rekening te brengen aan verkopers en consumenten. Hoofdstuk 3 suggereert dan dat het platform consumenten moet koppelen aan verkopers gebaseerd op voorkeuren om winsten te maximaliseren.

In hoofdstuk 4 worden retentie-aanbiedingen bestudeerd. Dit zijn verbeterde aanbiedingen die bedrijven doen aan consumenten die aangeven hun relatie met het bedrijf, bijvoorbeeld in de vorm van een abonnement of een bankrekening, te willen beëindigen.

Consumenten reageren op verschillende wijze op dergelijke praktijken. Sommige consumenten zijn actief op zoek naar een retentie-aanbod terwijl anderen vaak helemaal geen weet hebben van het bestaan van dergelijke aanbiedingen.

Hoofdstuk 4 bestudeert een setting waarin er twee types consumenten zijn, namelijk die met relatief lage, en die met relatief hoge overstapkosten. Retentie-aanbiedingen kunnen dan door bedrijven gebruikt worden als filtering-mechanisme. Consumenten die al de moeite hebben genomen een aanbieding van een concurrent te bemachtigen, geven het signaal af dat ze lage overstapkosten hebben en daarom makkelijk overstappen. Door middel van retentie-aanbiedingen kunnen bedrijven dus discrimineren op prijs.

Het hoofdstuk beschouwt dus situaties waarin consumenten hun huidige abonnement kunnen opzeggen ten faveure van dat van een concurrent. Als een consument bijvoorbeeld van zorgverzekering wil veranderen moet ze, voor de daadwerkelijke overstap, de huidige opzeggen. Dit biedt haar huidige verzekeraar de gelegenheid een verbeterd aanbod te doen.

Het model in hoofdstuk 4 bestaat uit 2 periodes waarin 2 bedrijven gedifferentieerde producten verkopen. In periode 1 vind er gewone concurrentie plaats. In periode 2, daarentegen, kunnen bedrijven drie verschillende prijzen vragen: een prijs voor loyale consumenten, een prijs voor overstappers, en een retentie-prijs voor consumenten die de intentie hadden over te stappen maar toch zijn gebleven.

Wat hoofdstuk 4 uniek maakt in vergelijking met de literatuur is dat consumenten strategisch kunnen handelen door een overstap te simuleren om zodoende een retentie-aanbod te verkrijgen. Een dergelijke simulatie kost wel wat, bijvoorbeeld in de vorm van moeite, tijd of geld. Consumenten met lage overstapkosten zullen het toch de moeite waard vinden om een dergelijke actie uit te voeren. Dit biedt bedrijven de mogelijkheid tussen consumenten met verschillende overstapkosten te prijsdiscrimineren.

De analyse van hoofdstuk 4 suggereert dat de mogelijkheid van retentie-aanbiedingen leidt tot hogere prijzen. De prijs voor loyale consumenten stijgt omdat zij gemiddeld genomen niet snel overstappen. De prijzen die bedrijven gebruiken om consumenten te stelen van de concurrentie stijgen echter ook. Dit wordt veroorzaakt door het strategisch handelen van sommige consumenten die al vast een deel van de overstap in gang te zetten, ook al hebben ze niet de intentie om over te stappen. Ook suggereert het hoofdstuk dat de prijzen stijgen voor consumenten die nog aan geen enkel bedrijf gebonden zijn. Dit komt omdat de competitie voor deze consumenten in periode 2 heviger wordt omdat enkelen van hen al een overstap in gang zetten, ook al zijn ze niet voornemens de overstap daadwerkelijk te maken. De mogelijkheid van het doen van retentie-aanbiedingen heeft een ambigu effect op de welvaart. Bedrijven kunnen hogere winsten behalen terwijl alle consumenten tezamen slechter af zijn.

Hoofdstuk 5 bestudeert de trouwmarkt: een markt met substantiële overstapkosten wanneer men van partner wisselt. Om precies te zijn, hoofdstuk 5 onderzoekt of er een relatie bestaat tussen ongeduld en huwelijkse ontrouw; de eerste stap in het wisselen van partner.

Vreemdgaan is een onderwerp van een niet te onderschatten belang. Overspel kan leiden tot een scheiding met bijbehorende economische consequenties zoals het verlies van inkomen of schaalvoordelen. Daarnaast zijn er vanzelfsprekend ook niet-monetaire gevolgen. Zelfs als een relatie een buitenechtelijke affaire overleeft, heeft dit vaak tot gevolg dat de verstandhoudingen binnen het huwelijk zijn verstoord.

De bestaande literatuur negeert vaak het feit dat de consequenties van een buitenechtelijke affaire in de toekomst liggen en dat ze daarom gedisconteerd dienen te worden. Een overspelige weet namelijk dat zijn/haar partner mogelijkerwijs achter de affaire komt en hij/zij dan gestraft zal worden, en zal de consequenties wegen met een disconteringsfactor. Sommige artikelen gebruiken het scholingsniveau als proxy voor de disconteringsfactor. Dit niveau heeft echter ook invloed op de kans op overspel via andere kanalen.

Hoofdstuk 5 maakt gebruik van het ongeduld om de disconteringsfactor te benaderen en toont aan dat mensen die ongeduldiger zijn vaker een buitenechtelijke affaire hebben. Als proxy voor ongeduld wordt het rookgedrag van individuen gebruikt omdat er volgens de bestaande literatuur een sterk verband is tussen beide. Het hoofdstuk laat zien dat (voormalige) rokers vaker overspelig zijn.

Ook werpt het hoofdstuk nieuw licht op de relatie tussen het scholingsniveau en overspel. Educatie kan via drie wegen een effect hebben op de kans op vreemdgaan. Ten eerste heeft scholing een effect op de kwaliteit van een individu omdat beter geschoolde personen over het algemeen betere banen kunnen bemachtigen met bijbehorend salaris. Ten tweede heeft scholing een positief effect op de sociale vaardigheden van een persoon. Scholing vergroot via deze twee mechanismen de aantrekkelijkheid van een persoon en maakt een affaire aanlokkelijker. Ten derde hebben beter geschoolde personen gemiddeld genomen een lagere disconteringsfactor. Dit kan verklaren waarom overspel een negatieve relatie kan vertonen met opleidingsniveau zoals wordt gevonden in de bestaande literatuur. Hoofdstuk 5 vindt echter dat dit disconteringseffect van educatie (deels) wordt uitgefilterd als men rekening houdt met het ongeduldigheidsniveau van een persoon. Het effect dat educatie heeft op een persoon zijn aantrekkelijkheid wordt effectief gezien belangrijker waardoor de relatie tussen scholing en overspel positiever wordt. In sommige schattingen vind ik zelfs, in tegenstelling tot de literatuur, een positieve relatie tussen scholing en overspel.

Al met al geeft deze dissertatie interessante inzichten in de effecten van zoek en overstapkosten, eventueel in combinatie met andere economische fenomenen zoals loss aversion en retentie-aanbiedingen. Een belangrijke conclusie is dat bedrijfsactiviteiten die op het oog lijken bij te dragen aan het consumentensurplus, bijvoorbeeld retentie-aanbiedingen of het communiceren van product informatie, in werkelijkheid een negatief effect hebben op het welvaren van consumenten. Bedrijven profiteren juist van dergelijke activiteiten. Daarnaast blijken heterogene overstapkosten een relevante invloed te hebben op marktmechanismen. Hoofdstuk 5 toont empirisch de effecten van verschillende componenten van deze kosten, terwijl hoofstuk 4 aantoont dat deze heterogeniteit strategische mogelijkheden biedt. Ook mag de invloed van gedragsbiassen op marktwerking niet worden onderschat, zoals blijkt uit hoofdstuk 2 waar loss aversion de hoogte van prijzen beïnvloedt. Economisch beleid (voor markten met zoek- en overstapkosten) en de modellen waarop deze gestoeld wordt zou dus rekening moeten houden met al deze fenomenen alvorens conclusies te trekken.